## SOES 6006 Lecture 5

# 1-D Energy Balance Models

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# Overview : Lectures 4,5

1) Basics

- The global energy balance
- Zero-dimensional (globally-averaged) Energy Balance Models (EBMs)

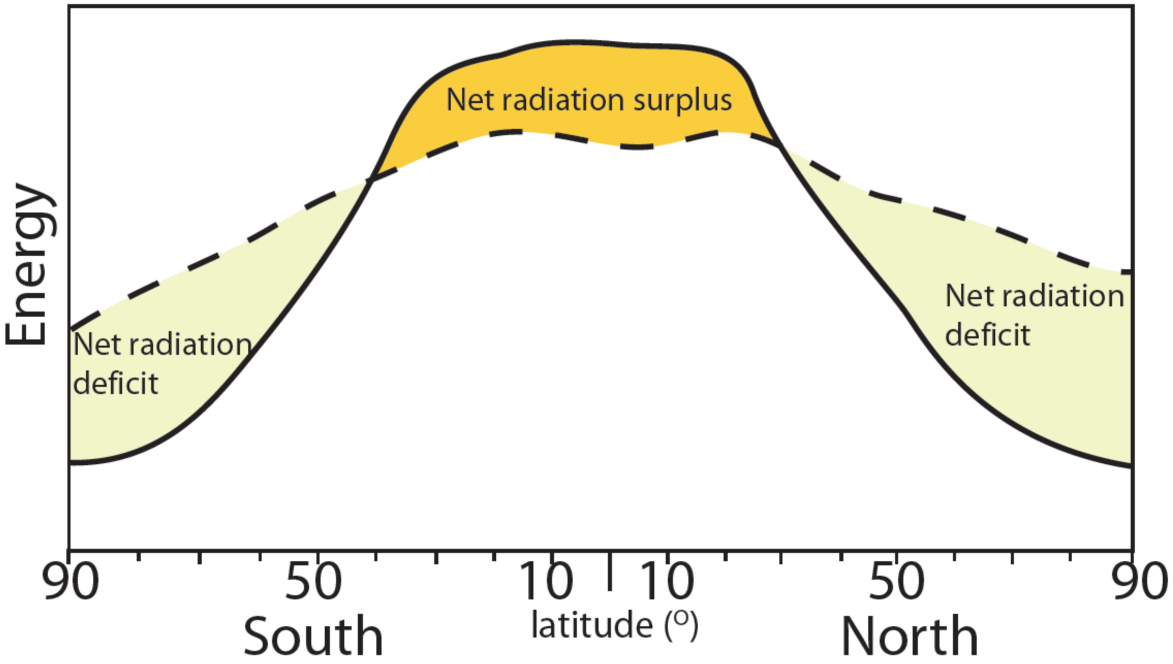
***2) Introducing latitudinal effects***

- ***1D (meridional) EBMs***
- ***Energy transport***

Themes (Lecture 5):

- Some spherical geometry, for ISWR as function of latitude
- A (slightly) more sophisticated OLWR formula
- Simplest representation of ocean/atmosphere heat transport
- ***Modelling zonally-averaged surface temperature***
- The ice-albedo feedback & hysteresis

# 1) Zonal radiation imbalance

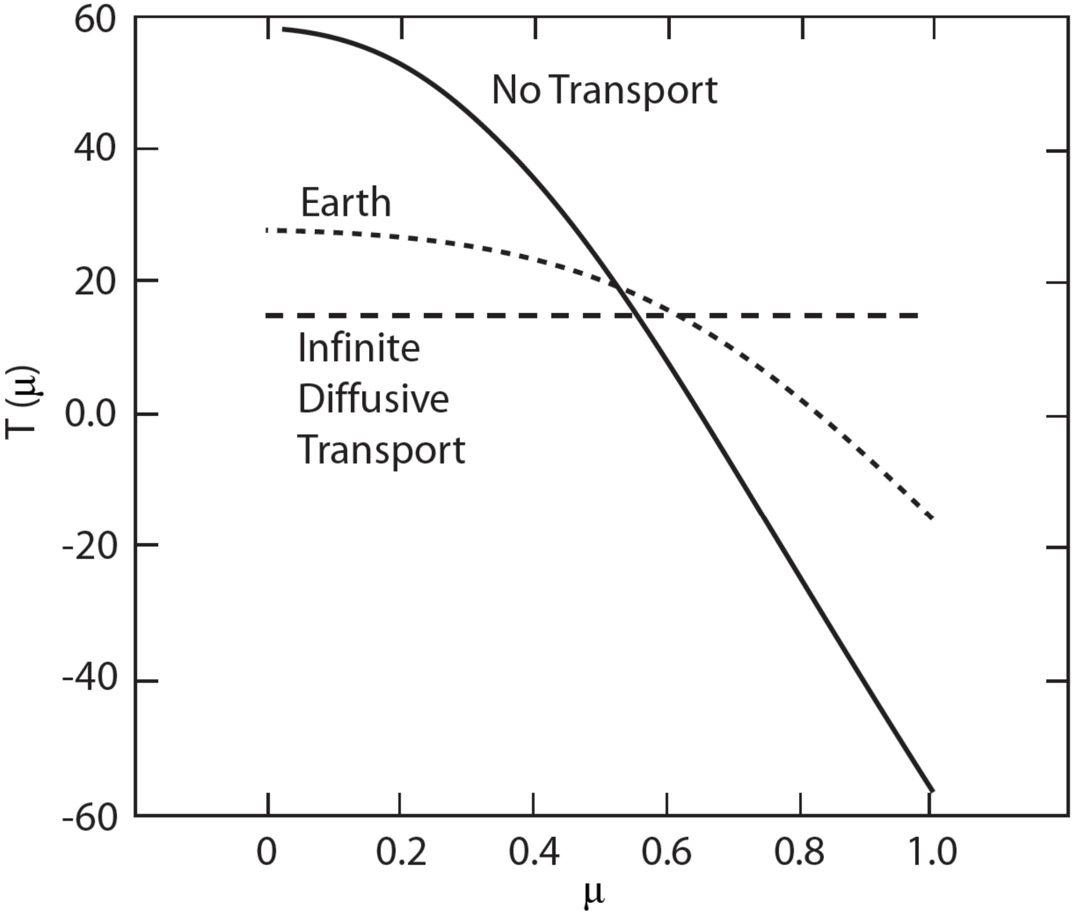


Distribution of absorbed solar and emited infrared radiation with latitude
Tropics = energy surplus
High Latitudes = deficit

***What factors govern the surplus & deficit of radiation?***

Schematic, for precise diagram go to: Kump, L.R., Arthur, M.A., Patzkowsky, M.E., Gibbs, M.T., Pinkus, D.S., Sheehan, P.M., (1999)
A weathering hypothesis for glaciation at high atmospheric pCO2 in the late Ordovician. Palaeoclimatol.Palaeoecol. Palaeogeogr. v, 152, p. 173-187

# 2) Meridional heat transport & meridional temperature gradient

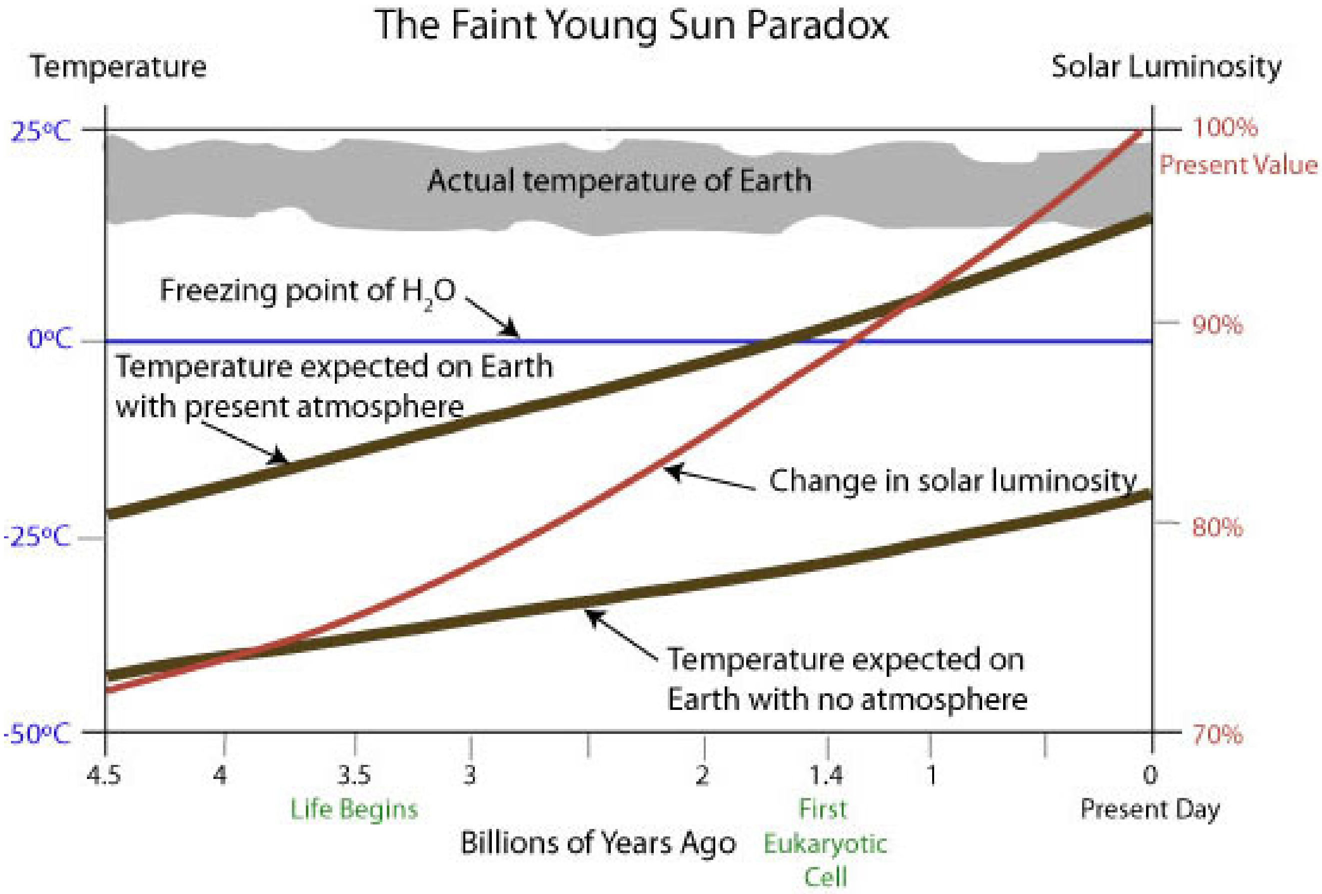


Surface T(K) vs. sine of latitude for the case of no transport, infinite transport, and earth. T is zonally averaged.

***How should we represent meridional transport of heat in a 1-D EBM?***

Reproduced by permission of American Geophysical Union: North, G.R., Cahalan, R.F., Coakley, J.A, jr. Energy Balance Climate Models. Rev. Geopys. Space Phys, v. 19, no. 1, p. 91-121. February 1981. Copyright [1981] American Geophysical Union.

# 3) The Faint Young Sun Paradox

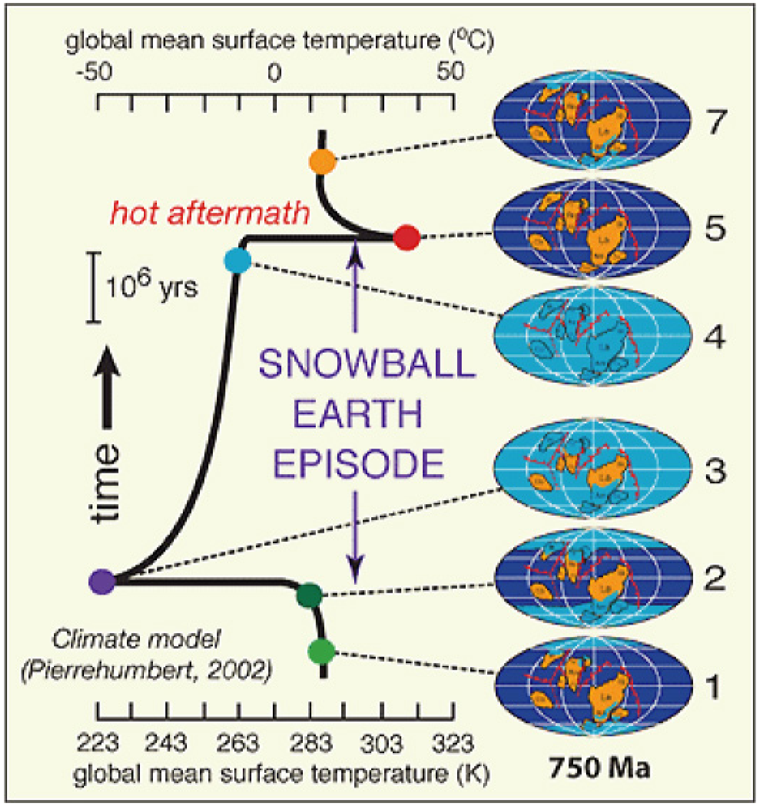


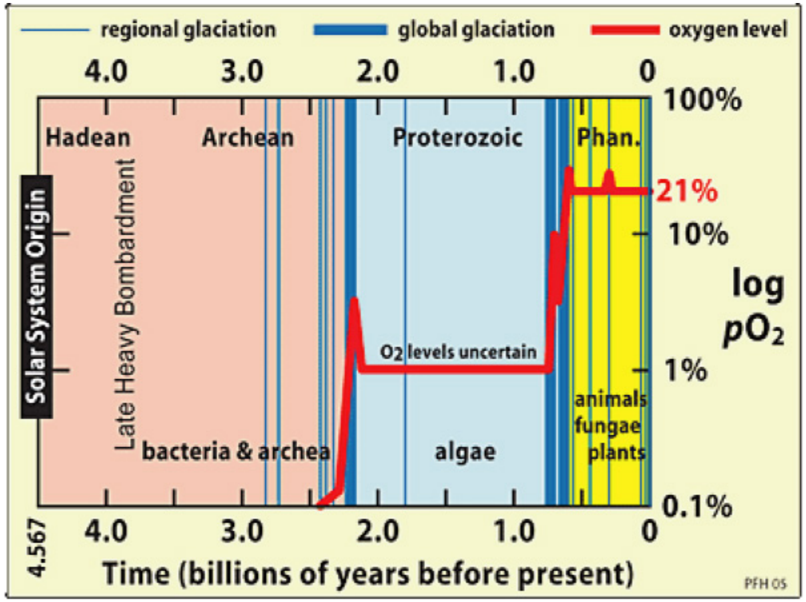
Even though the Sun was about 30% dimmer than it is now, the temperature on Earth has been more or less stable.

***How can 1-D EBMs help explain liquid water on the early Earth?***

Courtesy of University of Oregon

# 4) Snowball Earth Episodes





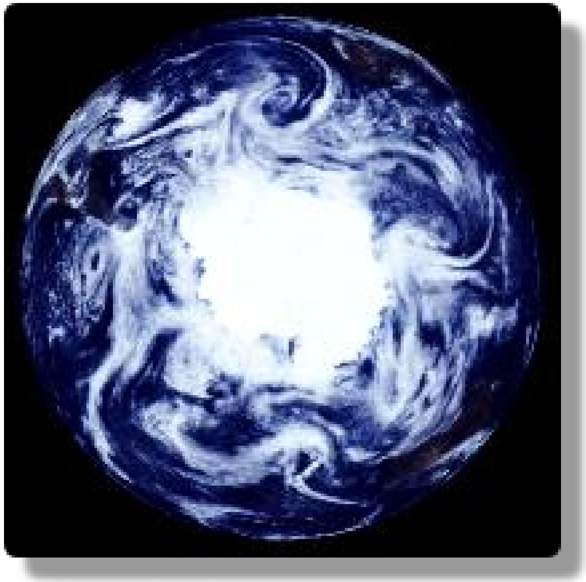
http://www.snowballearth.org/images/snwbltvst.gif

***Why were these episodes so long?***

Diagrams courtesy of Snowball Earth (Paul F Hoffmann).

# 5) Meridional heat transport in the Ocean & Atmosphere

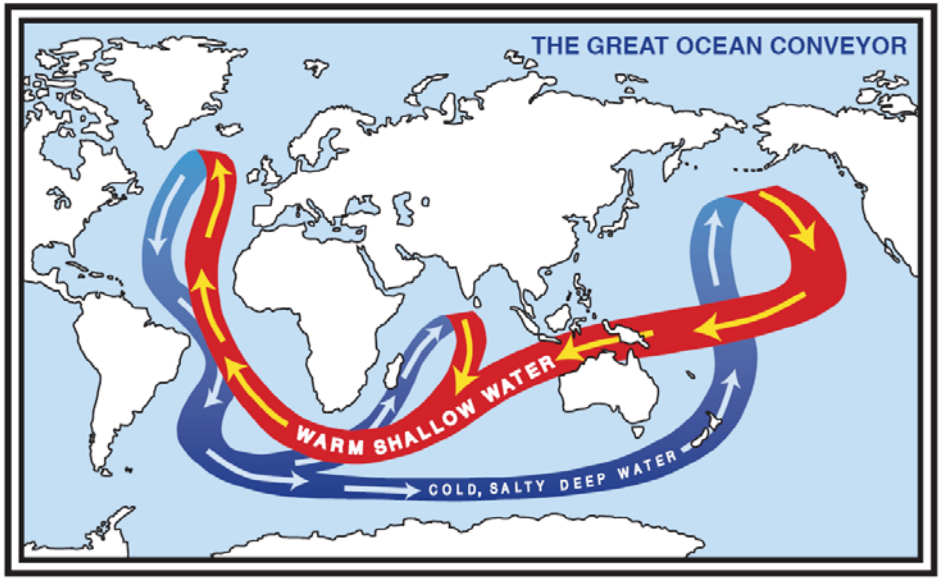
Courtesy of Center for Multiscale Modelling of Atmospheric Processes



Mid-latitude cyclones as viewed from space above the South Pole.

(http://www.cmmap.org/images/learn/climate/ferrel.jpg)

Courtesy of LDEO



The Global Ocean Conveyor Belt

***What is the difference between the transport represented in these two images, and how might it matter?***

# Meridional Non-uniformity

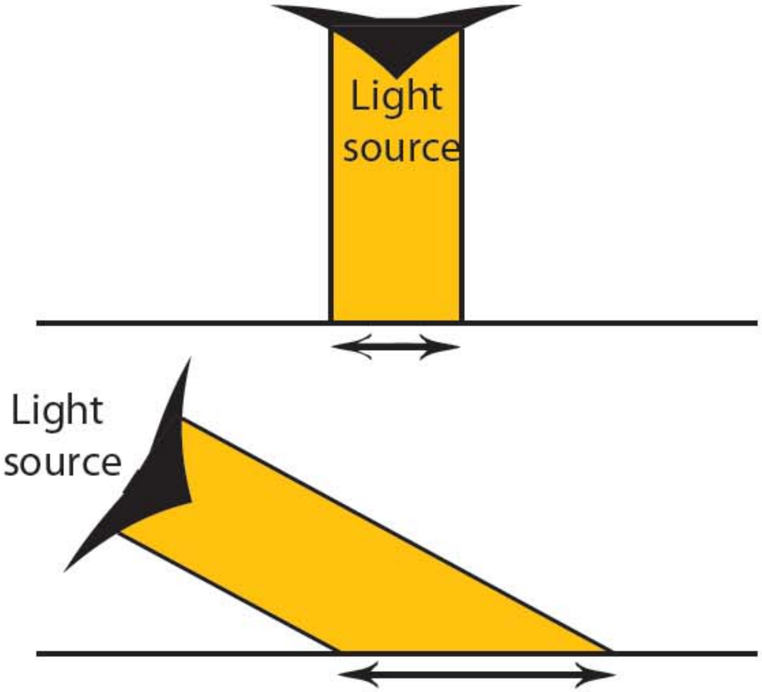
- ☐ Insolation varies with latitude
  - - allowing for the effect of obliquity
- ☐ Geometrical effects
  - - projected and actual areas of latitude bands
- ☐ Pole-Equator temperature differences
  - - moderated (reduced) by atmospheric & oceanic transports of heat (and also water, etc...)
  - - so need to include advection & diffusion , i.e. transport by mean circulation & eddies (recall end of lecture 3)
- ☐ We will build a more complete 1-D EBM …

# 1D (meridional) EBMs

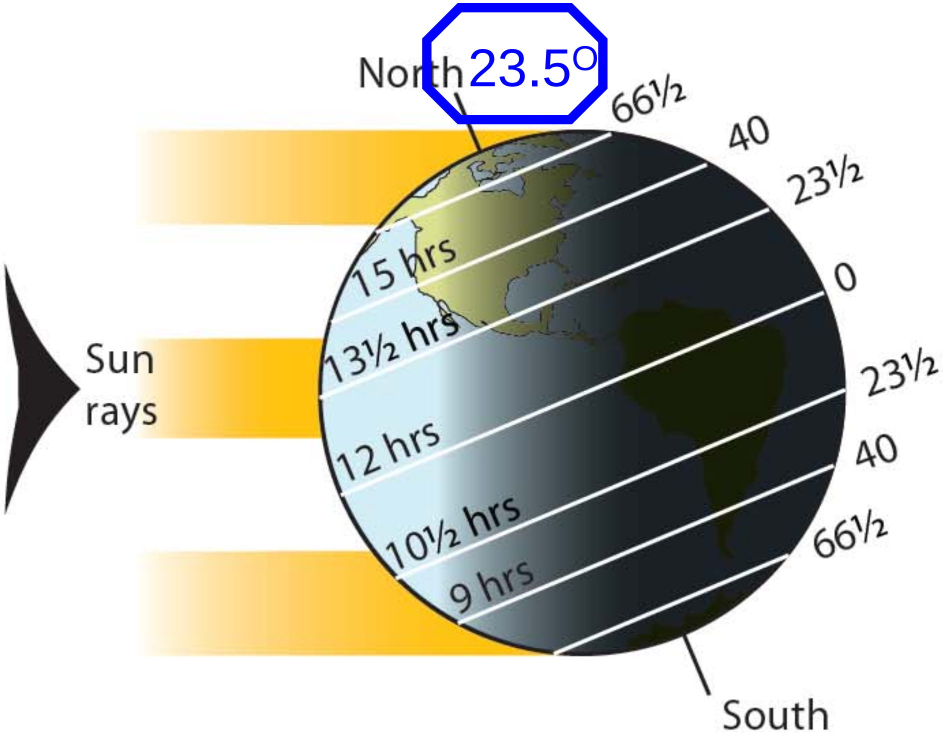
- Incoming SW solar radiation
  - varies as a function of latitude
  - and of albedo (may be a function of temperature...)
- Outgoing LW infra-red radiation
  - now a function of *local (not global)* surface temperature
- Heat transport, by mixing (and maybe advection)
  - needs to be parameterized
- Heat capacity of land/sea surface (mixed layer?)
  - needed for time-dependent calculations only
- See review by North et al (1981)
  - North GR, Cahalan RF & Coakley JA, Rev Geophys & Space Physics, 19, 91-121 (1981)

# ISWR at different latitudes

Angle of incidence affects the distribution of energy over the surface area (and also reflectivity of surface)



Present-day obliquity



# Projected Areas (Zones) for ISWR

Within latitude bands $(\theta - \Delta\theta/2)$ and $(\theta + \Delta\theta/2)$

*projected area :*

$$dA_p = 2R\cos\theta R\, d\theta \cos\theta \quad = \boxed{2R^2\cos^2\theta\, d\theta}$$

$$dA_p = \quad R^2(1+\cos 2\theta)d\theta$$

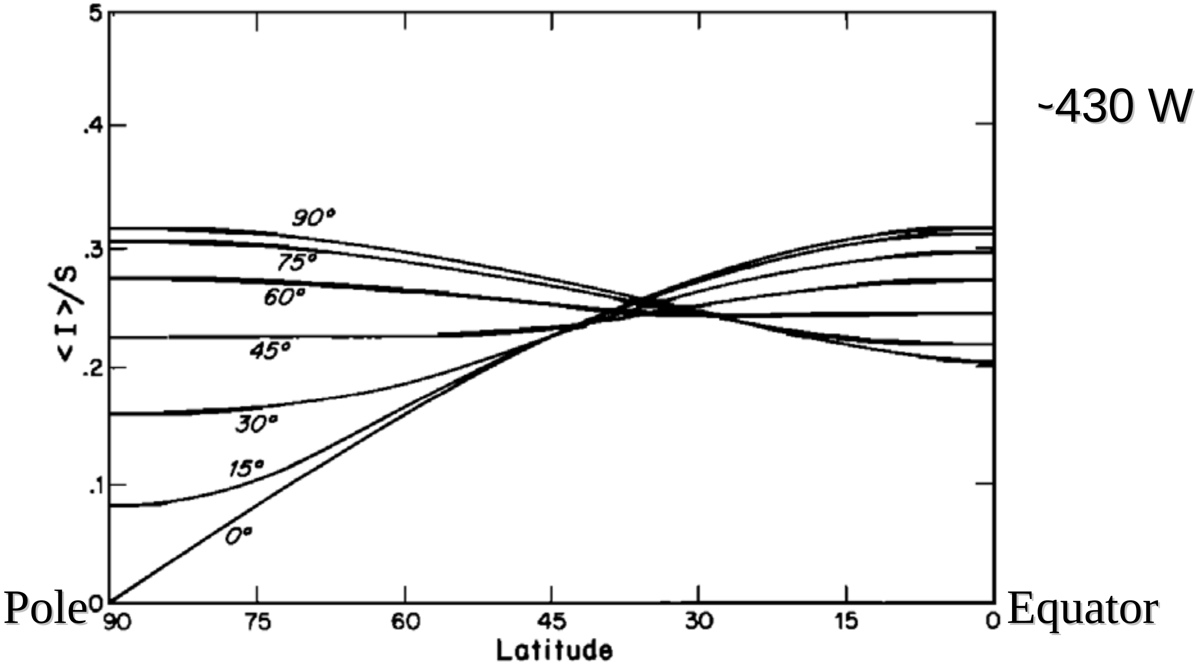
$$\therefore A_p = \int dA_p = R^2\left[\theta + \frac{1}{2}\sin\ 2\theta\right]_{\theta-\Delta\theta/2}^{\theta+\Delta\theta/2}$$

$$= R^2\left[\Delta\theta + \frac{1}{2}\left\{\sin\left(2\theta+\Delta\theta\right) - \sin\left(2\theta-\Delta\theta\right)\right\}\right]$$

$$\to \pi R^2/2 \ \ \textit{for a hemisphere} \ : \ OK$$

# Obliquity & *annual-mean* insolation

- For zero obliquity, ISWR varies as $\cos^2\theta$
- But non-zero Angle of Obliquity strongly alters the variation of annual-mean ISWR with latitude:



~430 Wm$^{-2}$ at equator

Fig. 9. Average annual insolation as a function of latitude for various values of the obliquity. The insolation is normalized to the solar constant at 1.52 AU.

Reproduced by permission of American Geophysical Union: Ward, W.R., Climatic variations on Mars, 1. Astronomical Theory of Isolation. Journal of Geophysical Research v 79, no 24. p, 3375. 20 August 1974. Copyright [1974] American Geophysical Union

# Annual-mean ISWR for present obliquity

- The annual-mean of seasonally-varying insolation, as a function of latitude, is complicated!
- But from observations (for present-day obliquity):

$$\boxed{S(\sin\theta) \approx S_{bar}\{1 - 0.477\,P_2(\sin\theta)\}}$$ (by G.R.North)

where $P_2(x) = (3x^2 - 1)/2$

(a ***second Legendre polynomial***)

- From which:

$S(Eq) = 340 \times \{1-(0.477\times(-0.5))\} = 340 \times 1.239 = 421\ Wm^{-2}$

vs. 433 W $m^{-2}$ for zero obliquity

$S(pole) = 340 \times \{1-(0.477 \times (-1))\} = 340 \times 0.523 = 178\ Wm^{-2}$

>> 0 for zero obliquity

- So the main effect of obliquity is polar warming

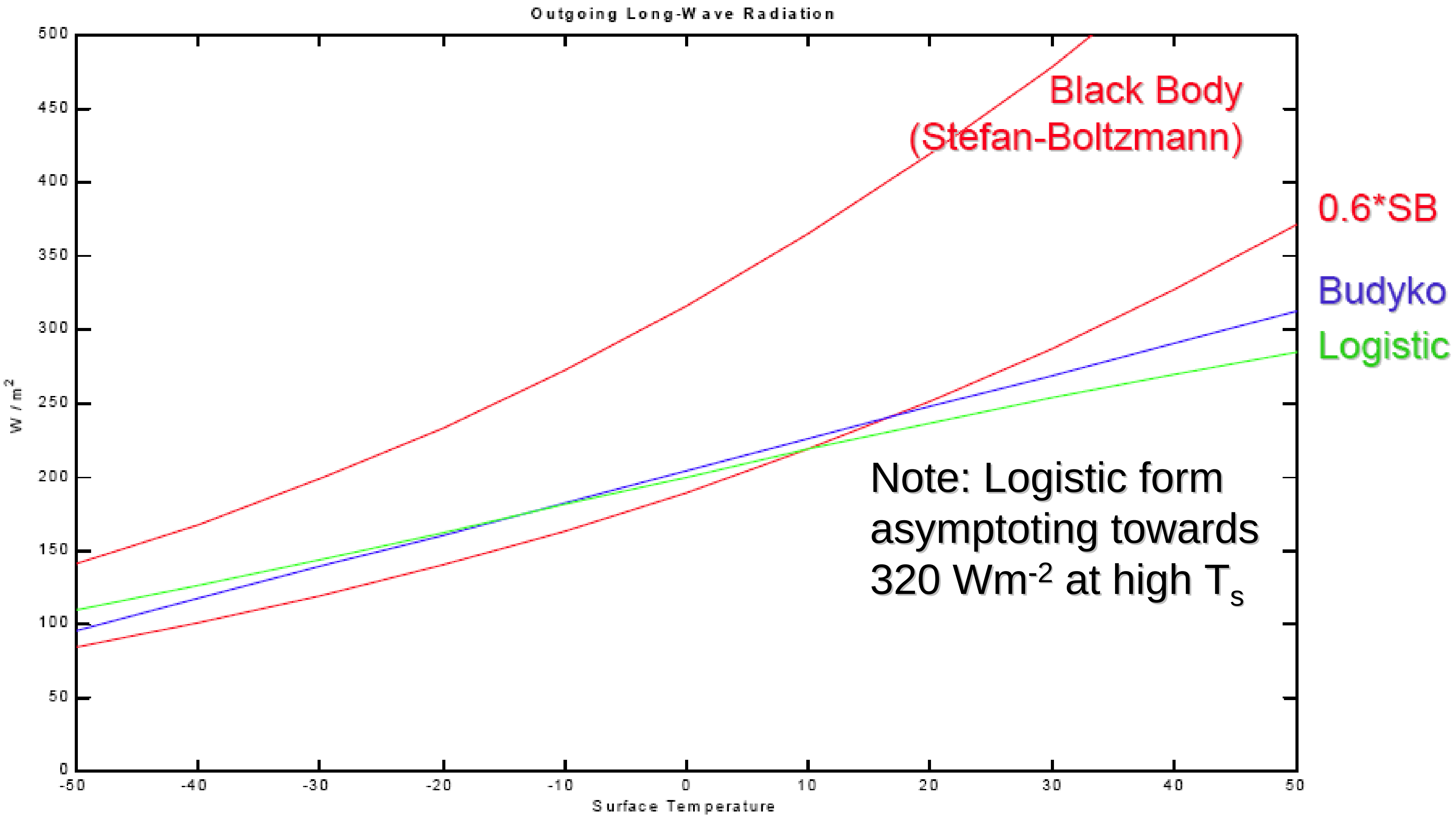
# OLWR re-visited

- ☐ Budyko's Linear Approximation for OLWR:
  - $F \approx 204 + 2.17\, T_s \qquad \mathrm{W\ m^{-2}}$
  - already (implicitly) incorporates the greenhouse effect due to water vapour
  - good approximation to data for $0 < T_s < 30$ ºC
  - but may be poor if extrapolated ***outside that range***
- ☐ A better approximation is the ***logistic form***:

$$\boxed{F \approx 0.9 \times \boldsymbol{\sigma}\, \mathbf{(}T_s{+}273)^4 \times [1\text{-logistic}\{(T_s\text{-}50)/100\}]}$$

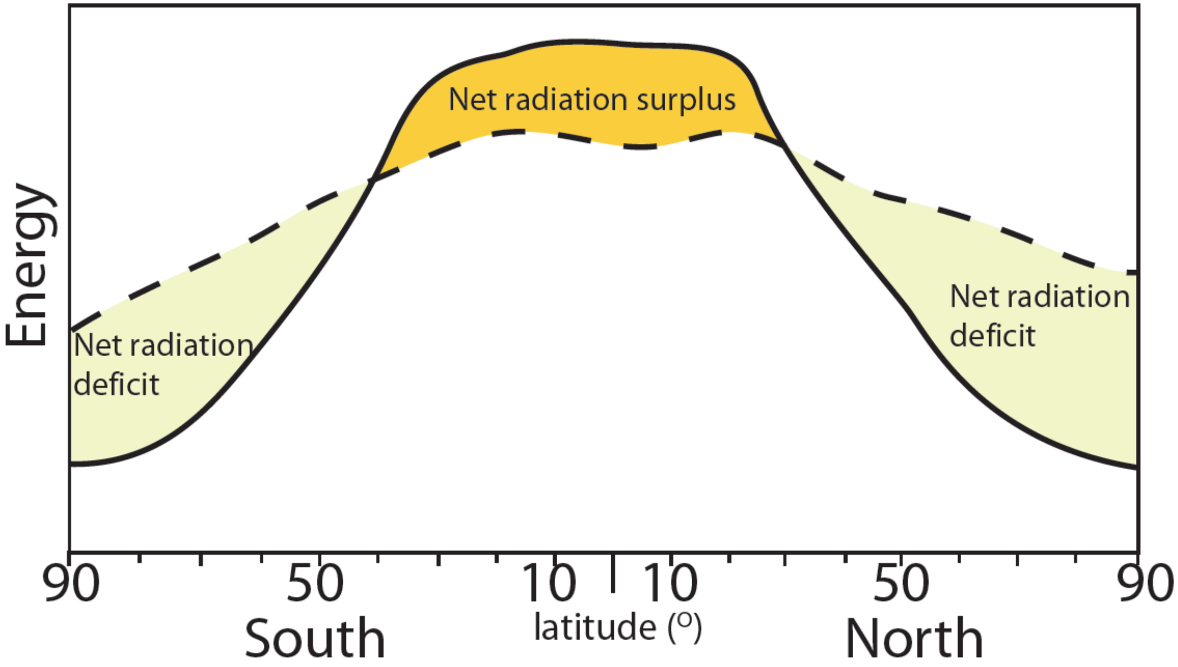
  - allows for ***saturation*** of $F$ (asymptoting to maximum of ~320 $\mathrm{Wm^{-2}}$) due to absorption by water vapour
  - recall last lecture (L4)

# Outgoing Long-Wave Radiation and the Water Vapour Greenhouse Effect



Note: Logistic form asymptoting towards 320 Wm$^{-2}$ at high $T_s$

# 1) Zonal radiation imbalance (explained)



Distribution of absorbed solar and emited infrared radiation with latitude
Tropics = energy surplus
High Latitudes = deficit

***Present-Day Obliquity & projected area effect ensure that considerably more ISWR received at low latitudes; OLWR largely determined by "surface temperature"***

Schematic, for precise diagram go to: Kump, L.R., Arthur, M.A., Patzkowsky, M.E., Gibbs, M.T., Pinkus, D.S., Sheehan, P.M., (1999)
A weathering hypothesis for glaciation at high atmospheric pCO2 in the late Ordovician. Palaeoclimatol.Palaeoecol. Palaeogeogr. v, 152, p. 173-187

# Atmospheric & Oceanic heat transport

- ☐ Latent & sensible heat transport
  - by advection (mean flow)
  - and/or by mixing (eddies), due to baroclinic instability

- ☐ Parameterize most simply (Budyko 1969):

$$\boxed{Q = K' (T_s - T_{bar})}$$

- relaxation of local temperature, $T_s$ to global mean temperature

  - useful for a simplest 1-D EBM, but results are peculiar …

# Heat Balance Equation - general form

- Evolution of temperature field (1D, meridional)

$$C\frac{\partial T_s(\theta)}{\partial t} = A(\theta)\left[(1-\alpha)S(\theta) - F(T_s)\right] - \frac{\partial Q}{\partial \theta}$$

**Net heating**

**ISWR**

**OLWR**

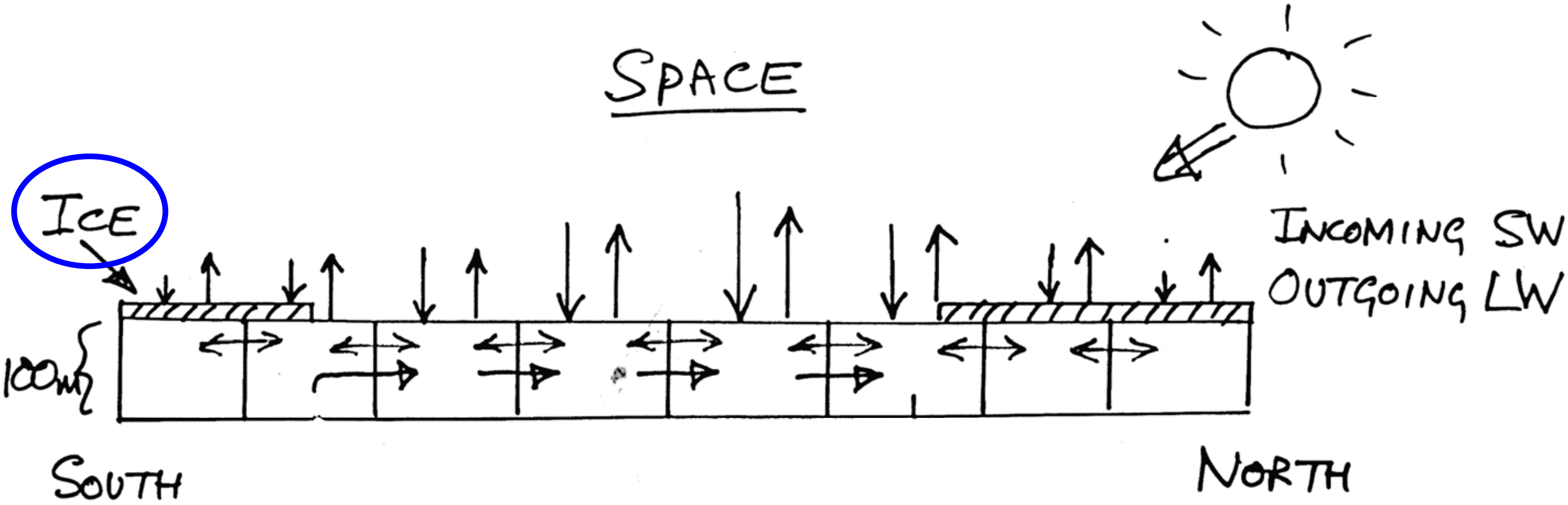
**Meridional divergence of transport**

$C$ = *heat capacity;*

$A(\theta)$ = *area of zone centred on latitude* $\theta$;

$Q$ = *advective-diffusive heat transport*

# One-dimensional (meridional) EBM schematic



(Sketch courtesy John Shepherd)

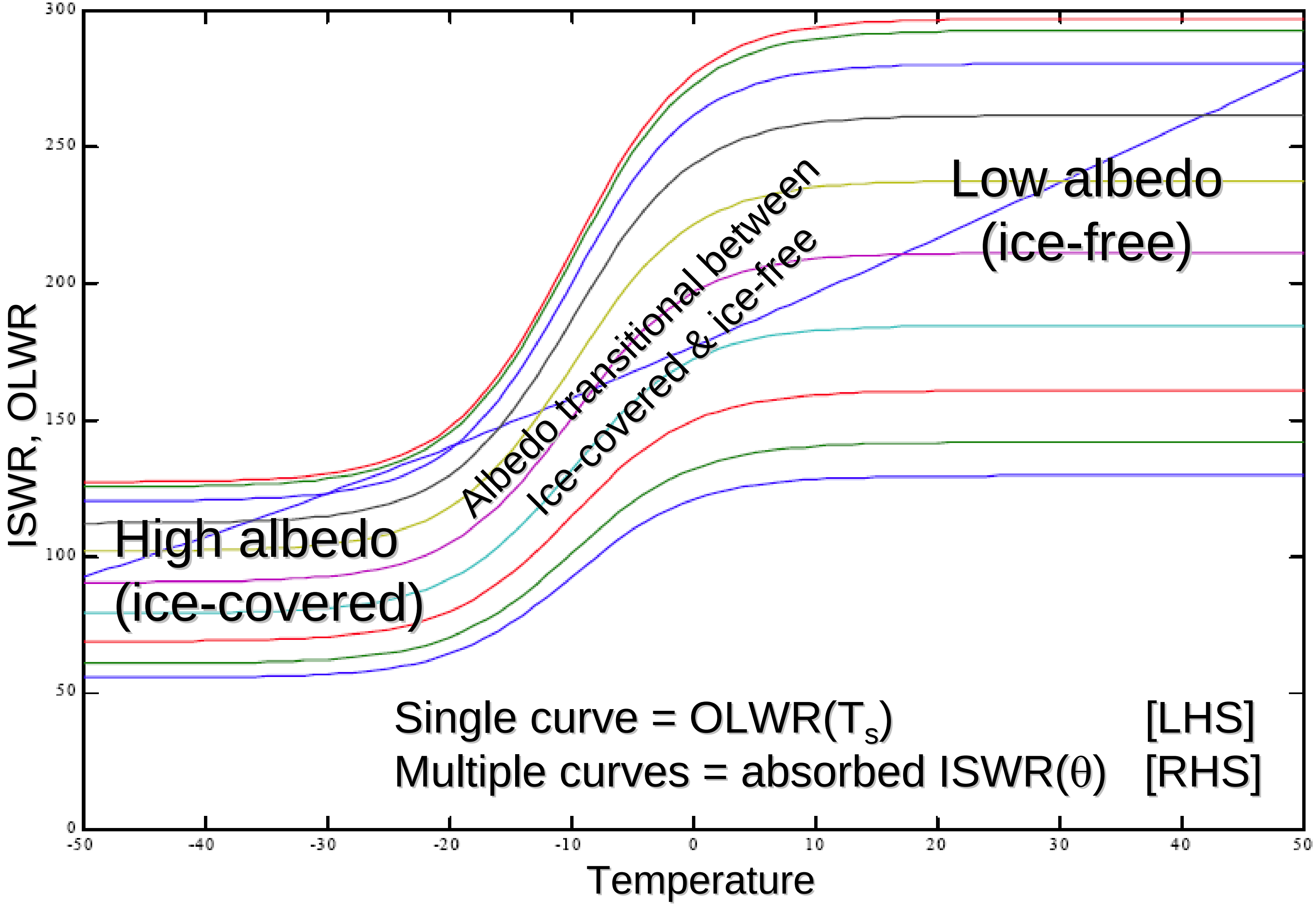
N.B. mixing between boxes (double-headed arrows) ***plus*** uni-directional transport (single-headed arrows) …

# Building a simple 1-D EBM

- Consider only the ***steady-state*** ($\partial T_s/\partial t = 0$)
- Use the logistic formulation for $F(\theta)$ - LHS below
- Use North's approximation for $S(\theta)$
- Optionally allow albedo to vary with latitude
  - i.e., ice-albedo feedback, via $T_s$
- Use Budyko's (simplest) formulation for mixing
  - therefore a ***local*** balance for each latitude band
- Put it all together as OLWR = ISWR + mixing:

$$
\begin{aligned}
&0.9\sigma T_s^4\left[1-\text{logistic}\{T_s - 50\}/100\right] \\
&\approx \left(1-\alpha(\theta)\right)\overline{S}\{1-0.477P_2(\sin\theta)\}-K'\left(T_s-\overline{T}\right)
\end{aligned}
$$

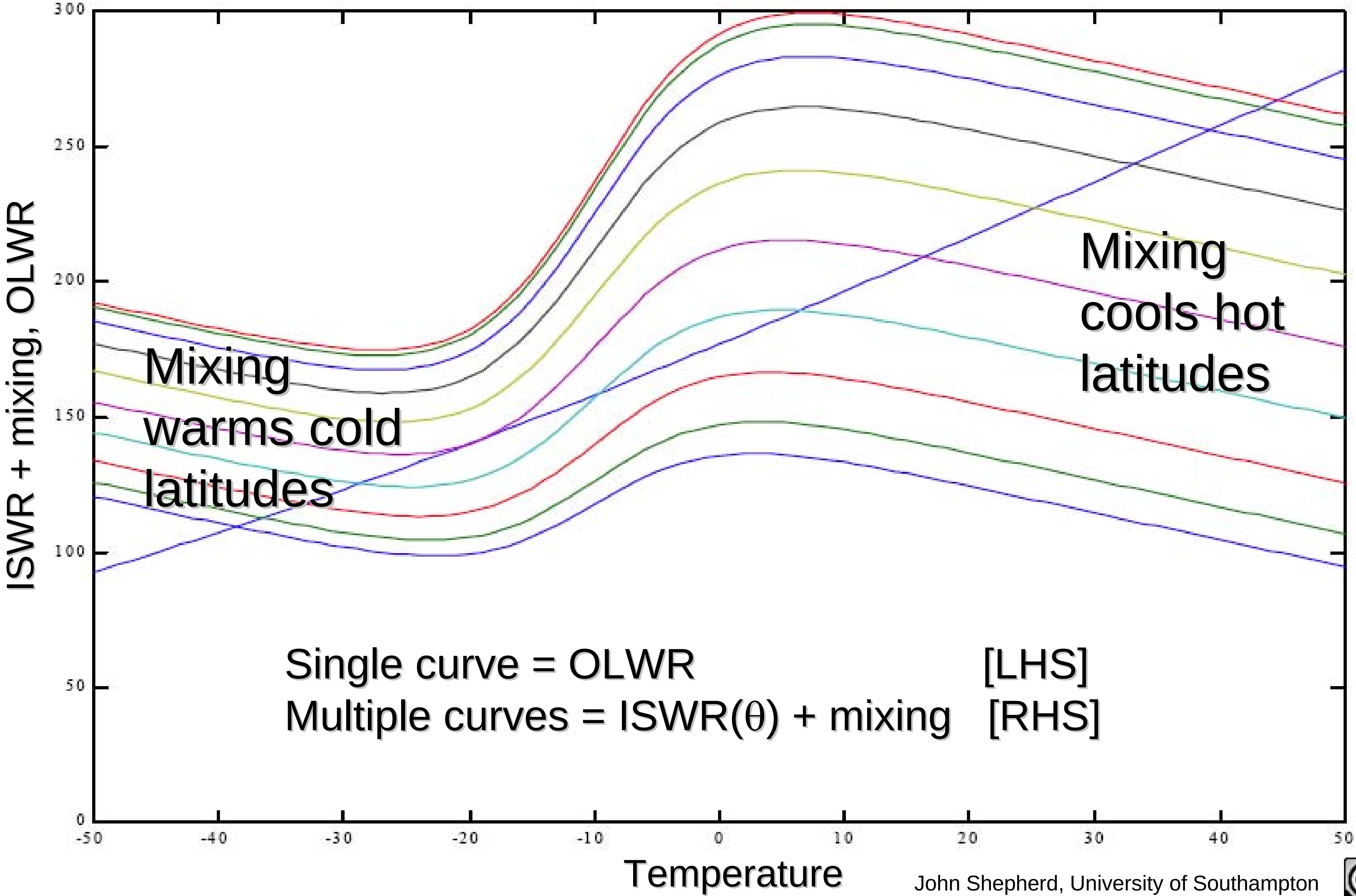
# 1-D EBM solutions - with Ice-Albedo and Water Vapour Feedbacks : Mixing = 0



Single curve = OLWR($T_s$) [LHS]

Multiple curves = absorbed ISWR($\theta$) [RHS]

# - with Ice-Albedo & Water Vapour Feedbacks, plus Budyko-type mixing, $Q = K'(T - T_{bar})$ [illustrative]

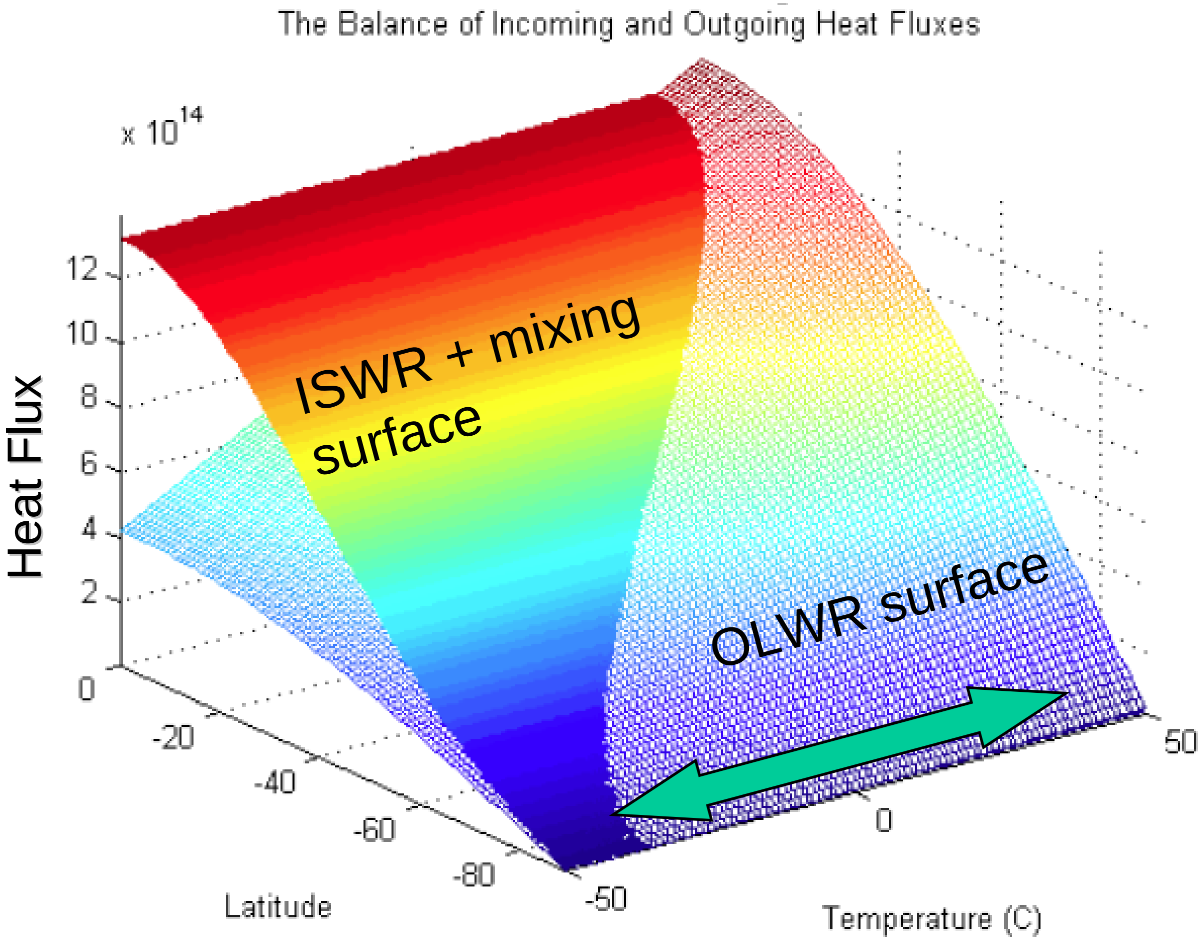


Mixing warms cold latitudes

Mixing cools hot latitudes

Single curve = OLWR [LHS]
Multiple curves = ISWR(θ) + mixing [RHS]

# 1-D EBM : Without Ice-Albedo Effect ; No Atmospheric/Ocean Mixing

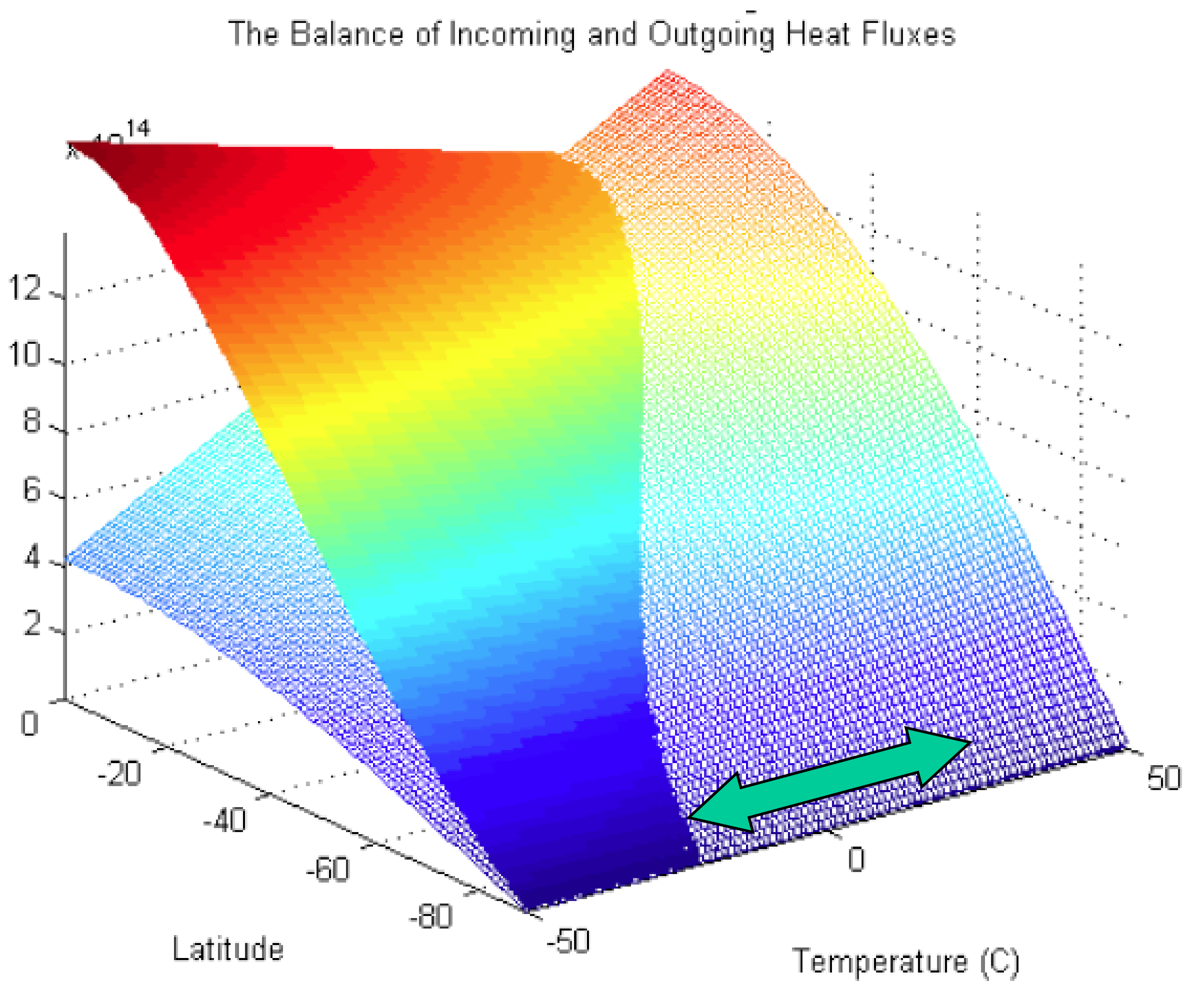


ISWR + mixing surface

OLWR surface

- “Solid” surface = ISWR + mixing (= 0); “Hatched” surface = OLWR
- Surfaces intersect (i.e., equlibria as function of latitude) where: Outgoing heat flux = Incoming heat flux

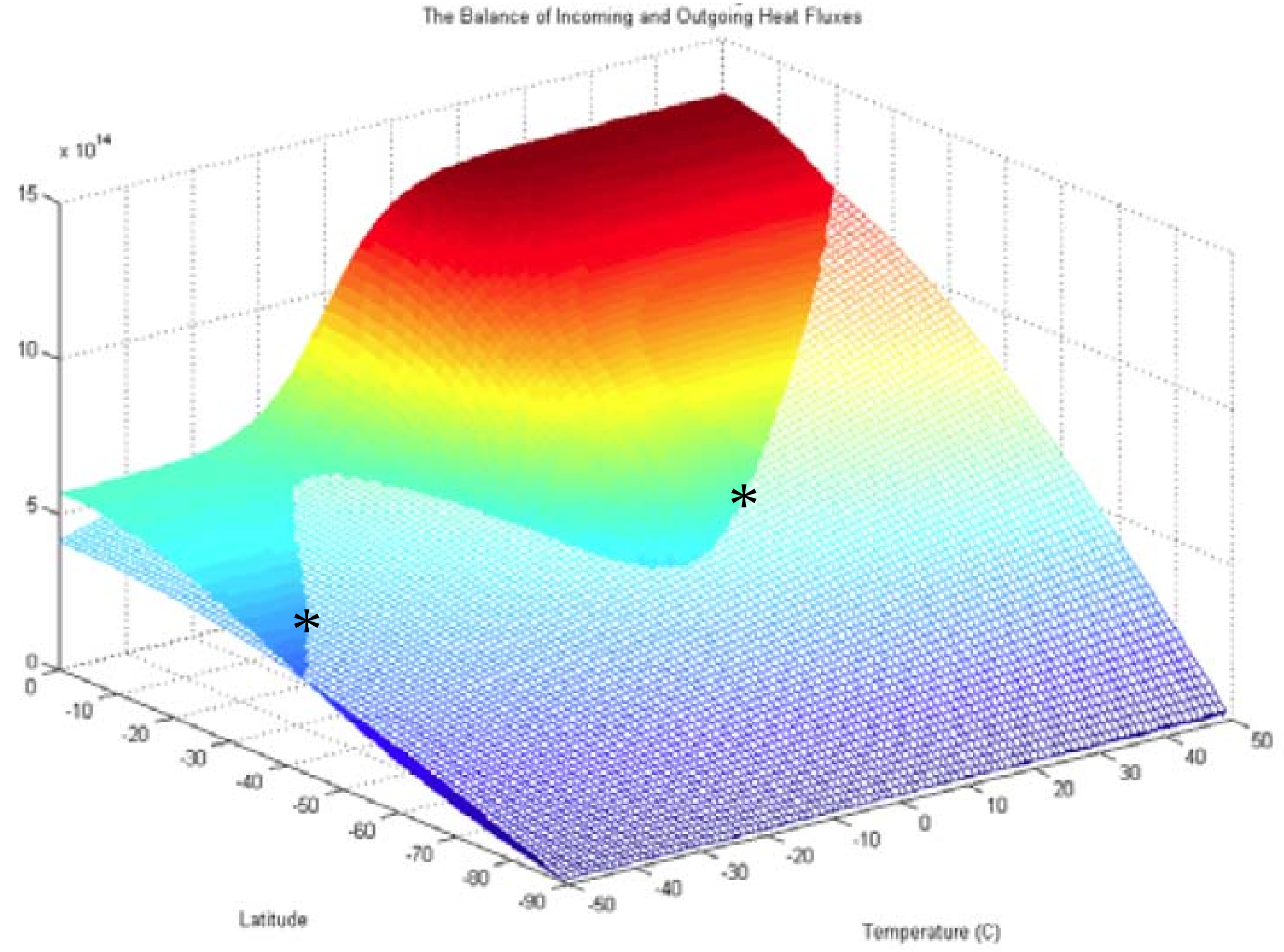
# 1-D EBM : Without Ice-Albedo Effect ; Mixing Diffusivity = 2 x $10^3$ mks



- Mixing reduces Equator-Pole temperature difference

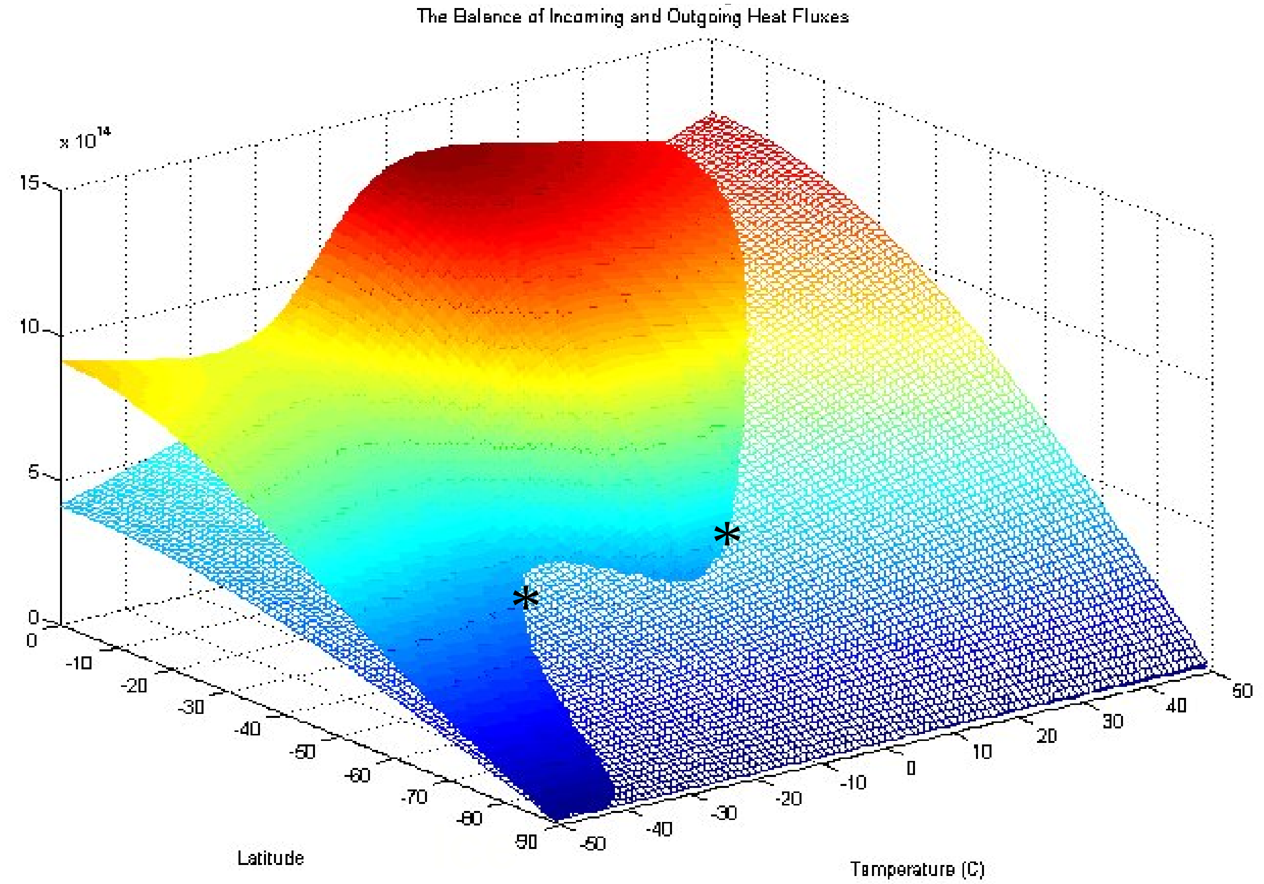
John Shepherd, University of Southampton

# 1-D EBM : ***With Ice-Albedo*** & Water Vapour Feedbacks ; Mixing = 0



- "Kink" in the surface reveals 2 stable states (e.g., *) possible across mid-latitudes (~15-40º) - supports possibility of "Snowball Earth"

# 1-D EBM : With Ice-Albedo and Water Vapour Feedbacks ; Mixing Diffusivity = 2 x $10^3$ mks



- Kink (hence bistability) reduced by meridional mixing
- Range of latitude with 2 stable states limited to ~50-60°

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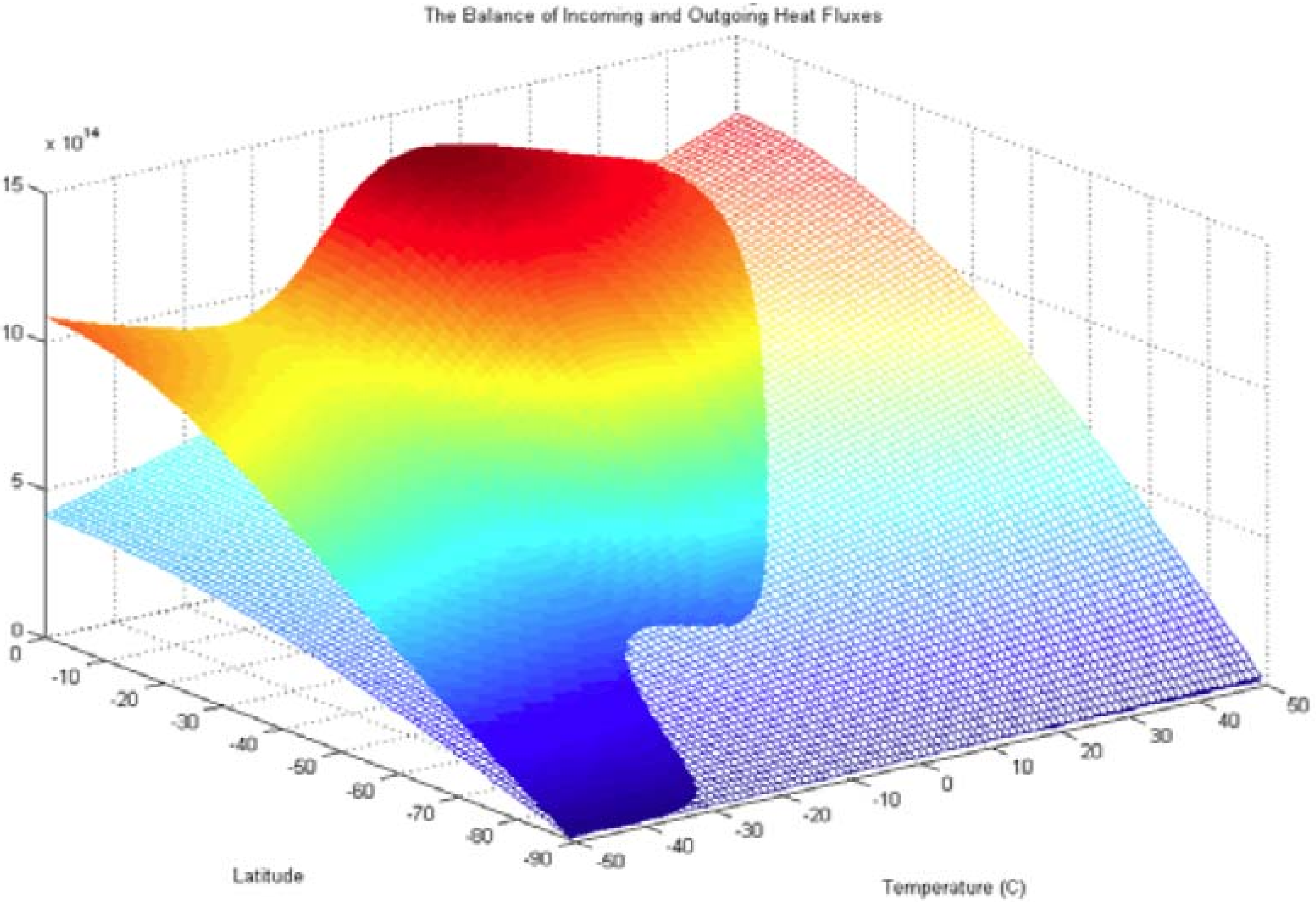
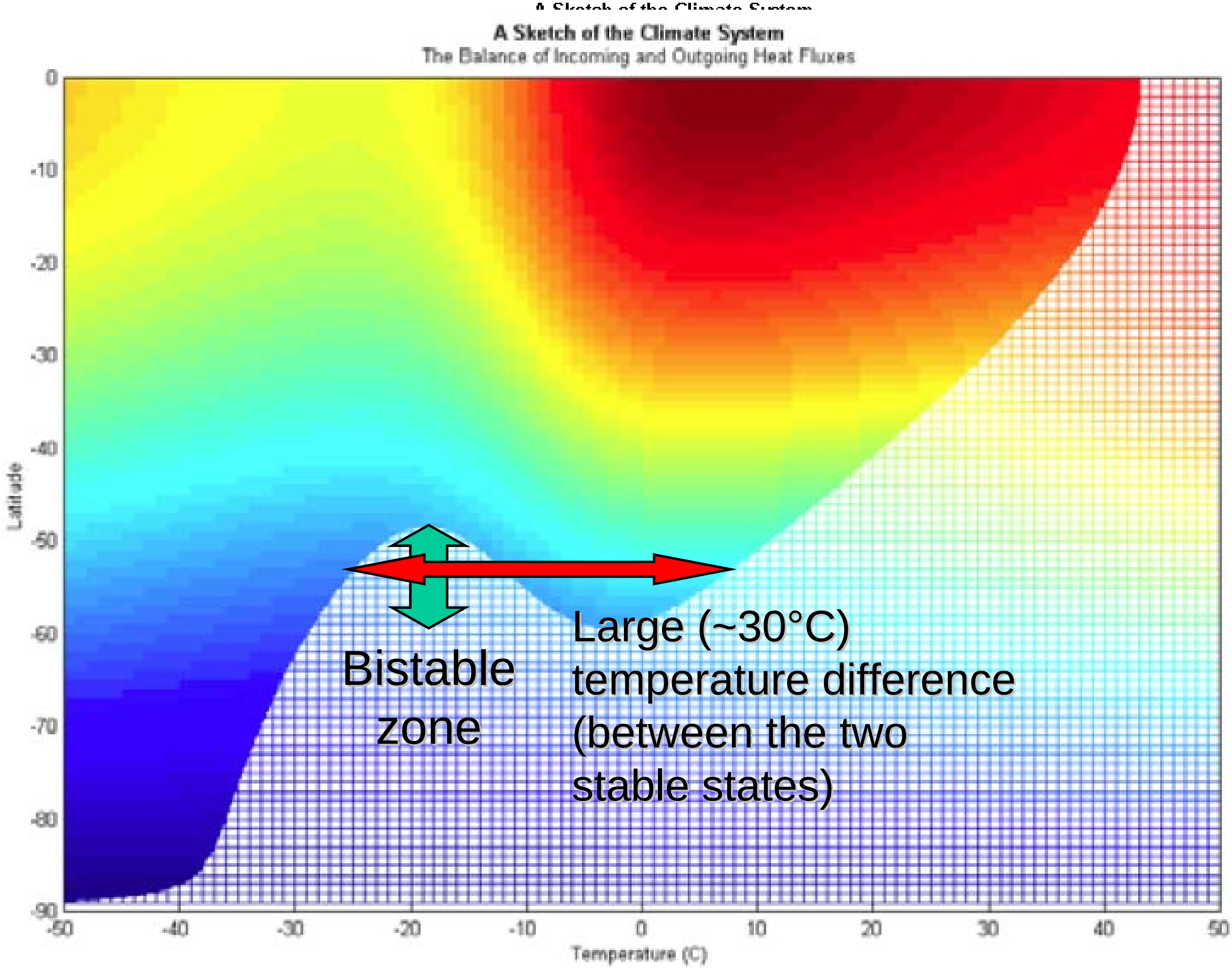
# 1-D EBM : With Ice-Albedo and Water Vapour Feedbacks ; (larger) Mixing Diffusivity = 3 x $10^3$ mks



- Kink almost eliminated under ***strong*** meridional mixing

# Plan view: With Ice-Albedo and Water Vapour Feedbacks ; Diffusivity = $2 \times 10^3$ mks



Bistable zone

Large (~30°C) temperature difference (between the two stable states)

# Incorporating proper mixing into 1-D EBMs

- ☐ *Fickian* diffusion, relating meridional mixing of heat to local temperature gradient:

$$\boxed{Q = K\, \partial T_S / \partial y}$$

- ☐ Several methods are available:

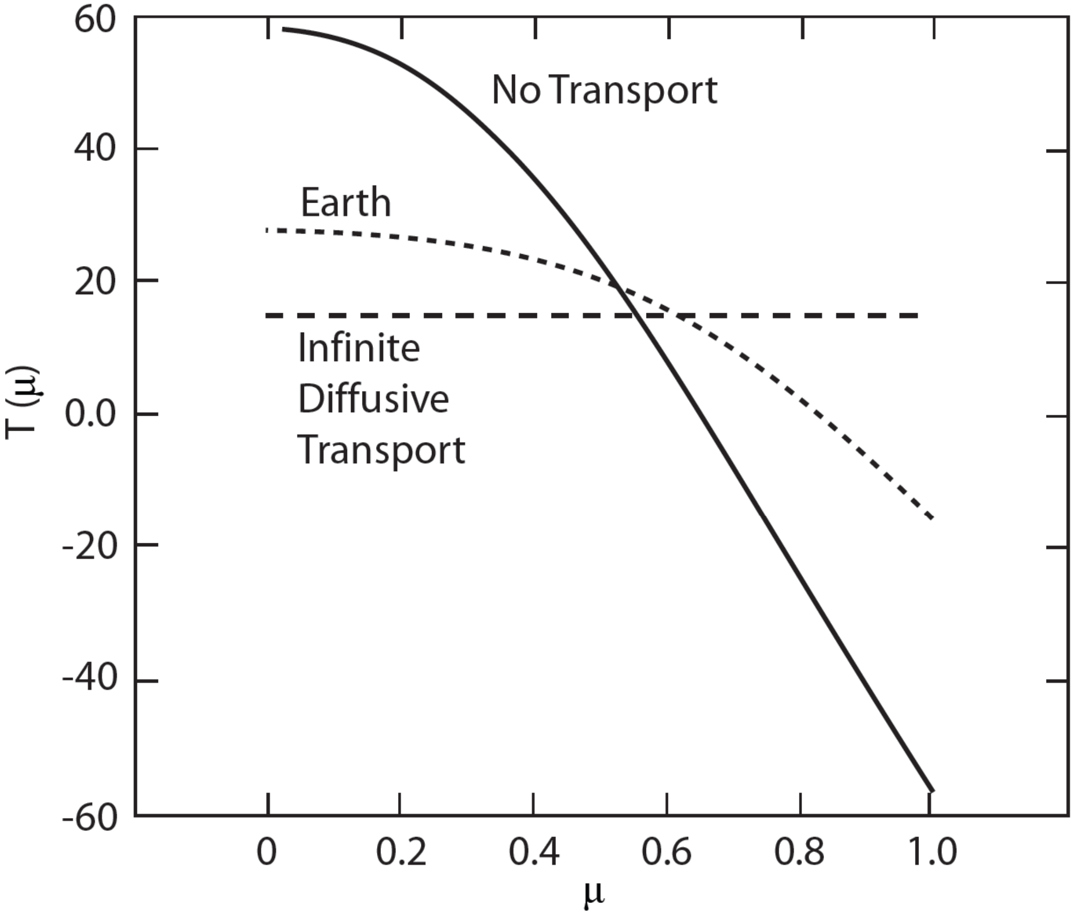
  (a) Analytical solutions, using expansion in Legendre polynomials in $sin(\theta)$ (North 1975, 1981)

  (b) Numerical solutions

  - ***steady-state*** : iterative solution of linear equations
  - ***time-dependent*** : solving first-order ordinary or partial differential equations

- ☐ And $K$ may be *constant or variable …*
  - e.g., parameterization of atmospheric eddy mixing (Peter Stone, MIT):
  - $\boldsymbol{K \propto k\, (\partial T / \partial y)^n}$ where 0.5 < n < 3 (varies with lat.)

# 2) Meridional heat transport & meridional temperature gradient (explained)



Surface T(K) vs. sine of latitude for the case of no transport, infinite transport, and earth. T is zonally averaged.

***A sensible parameterization is necessary, to reproduce the observed temp. gradient - e.g., Fickian mixing with appropriate choice of K***

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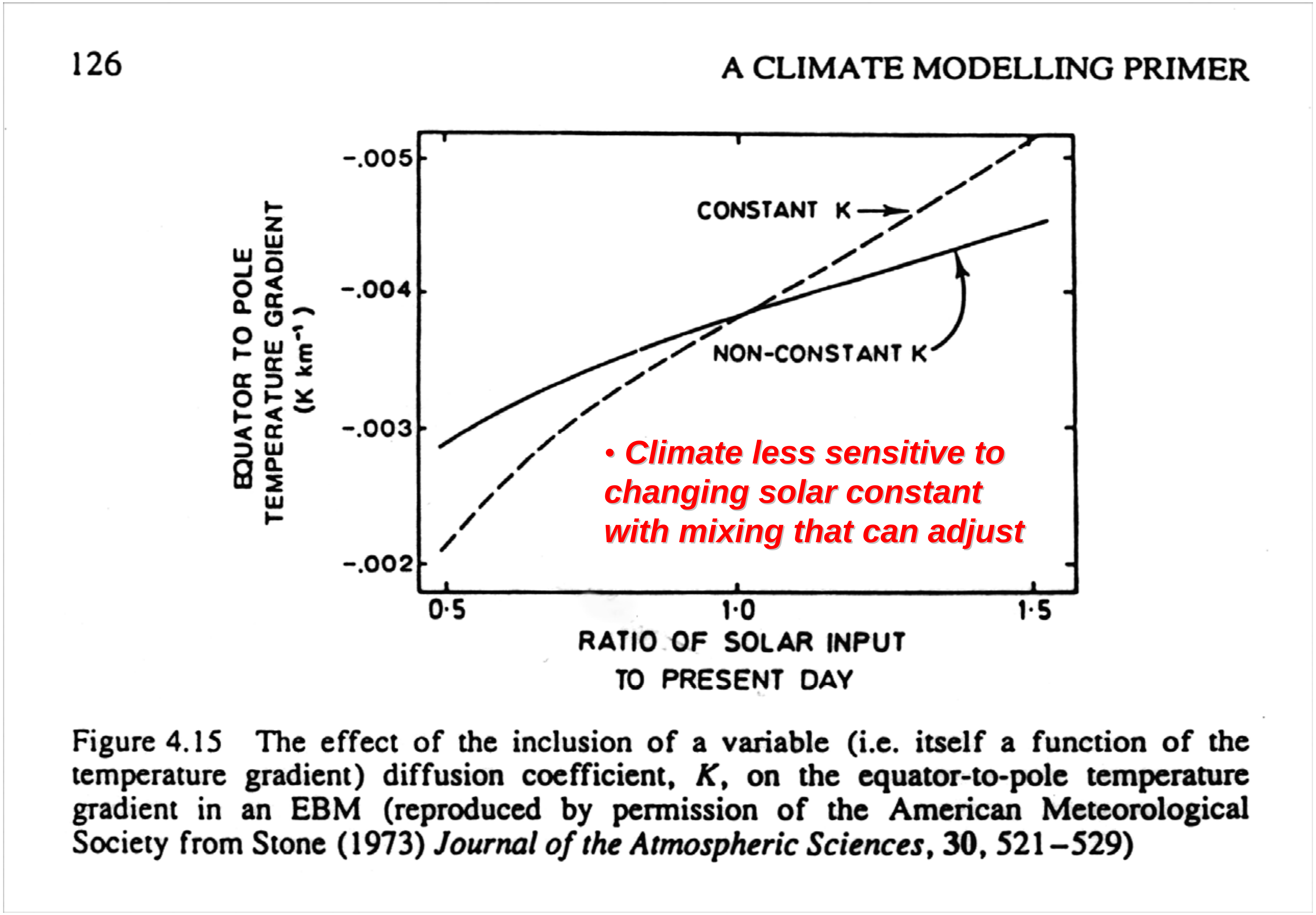
# Pole-Equator Temperature Gradient : vs. Solar Input & for constant / variable mixing

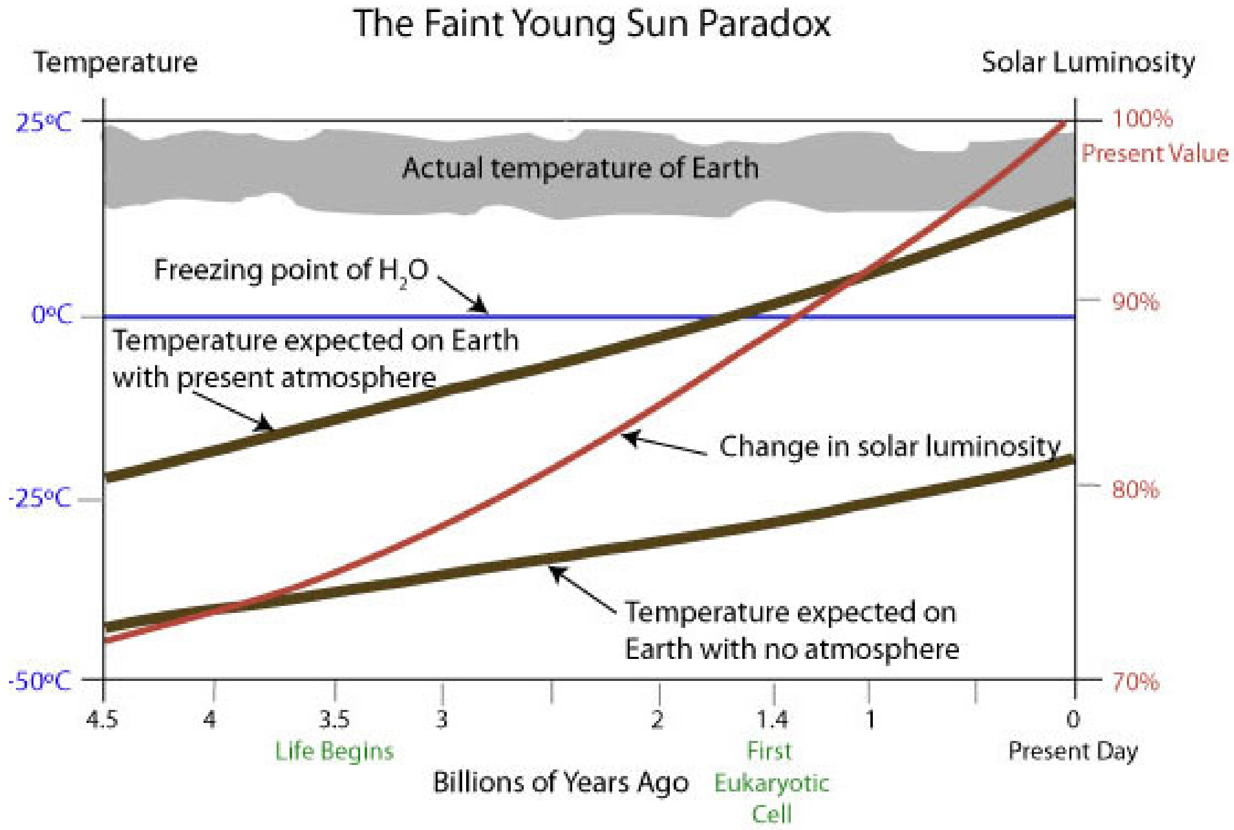


126 A CLIMATE MODELLING PRIMER

- ***Climate less sensitive to changing solar constant with mixing that can adjust***

Figure 4.15 The effect of the inclusion of a variable (i.e. itself a function of the temperature gradient) diffusion coefficient, *K*, on the equator-to-pole temperature gradient in an EBM (reproduced by permission of the American Meteorological Society from Stone (1973) *Journal of the Atmospheric Sciences*, **30**, 521–529)

# 3) The Faint Young Sun Paradox: Explained?

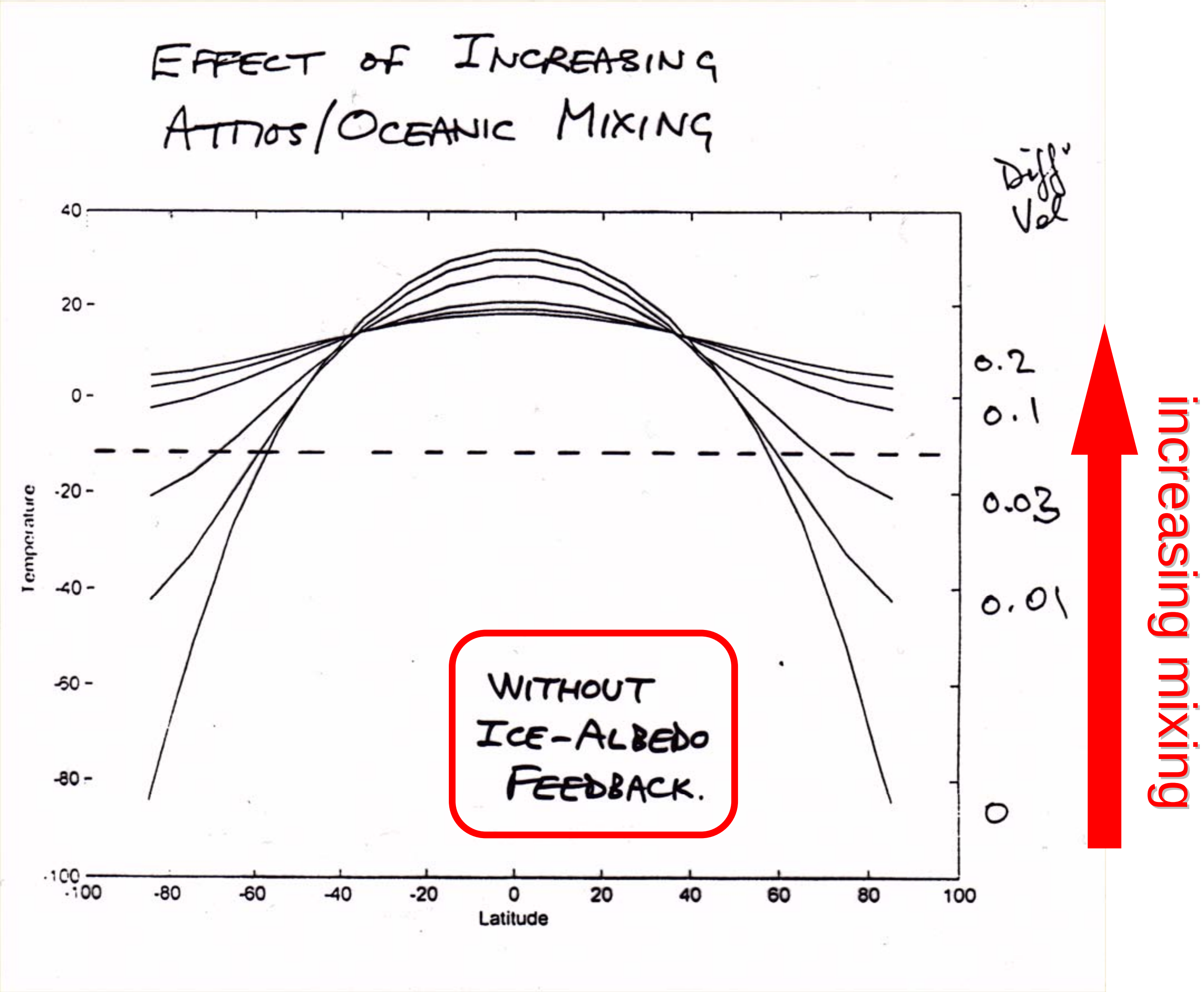

The Faint Young Sun Paradox

Even though the Sun was about 30% dimmer than it is now, the temperature on Earth has been more or less stable.

http://zebu.uoregon.edu/~imamura/122/images/faint_young_sun.jpg

***Could mixing help to keep poles (hence whole Earth) warm at lower luminosity? More likely associated with changing atmospheric composition***

# Simple EBM: the effect of (Fickian) mixing



increasing mixing

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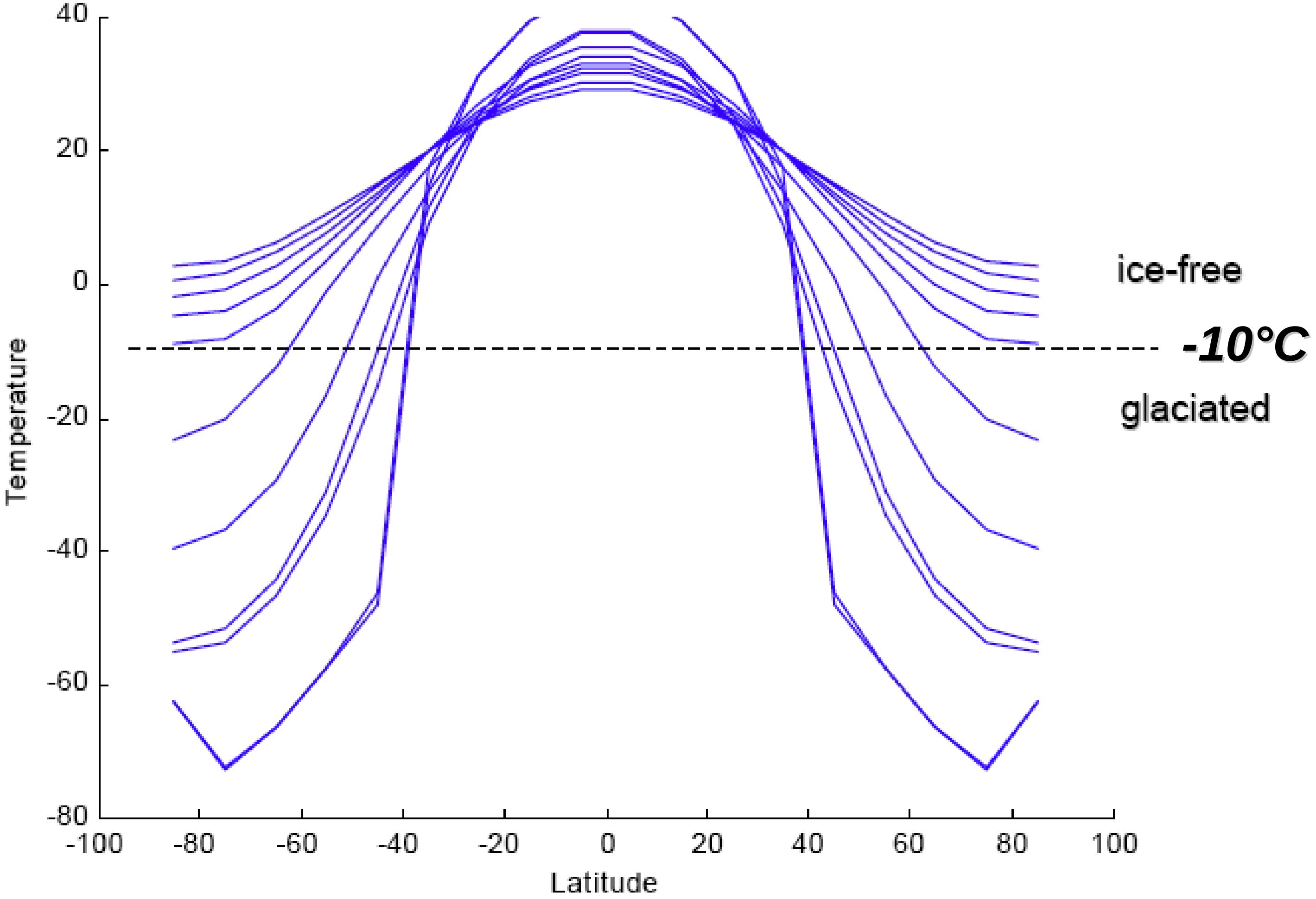
# 1-D EBM : ***with Ice-Albedo feedback*** ; ***variable mixing*** in the range 0 to 6 x $10^4$ $m^2s^{-1}$

(decreasing, then increasing, in increments of 1 x $10^4$ $m^2s^{-1}$)



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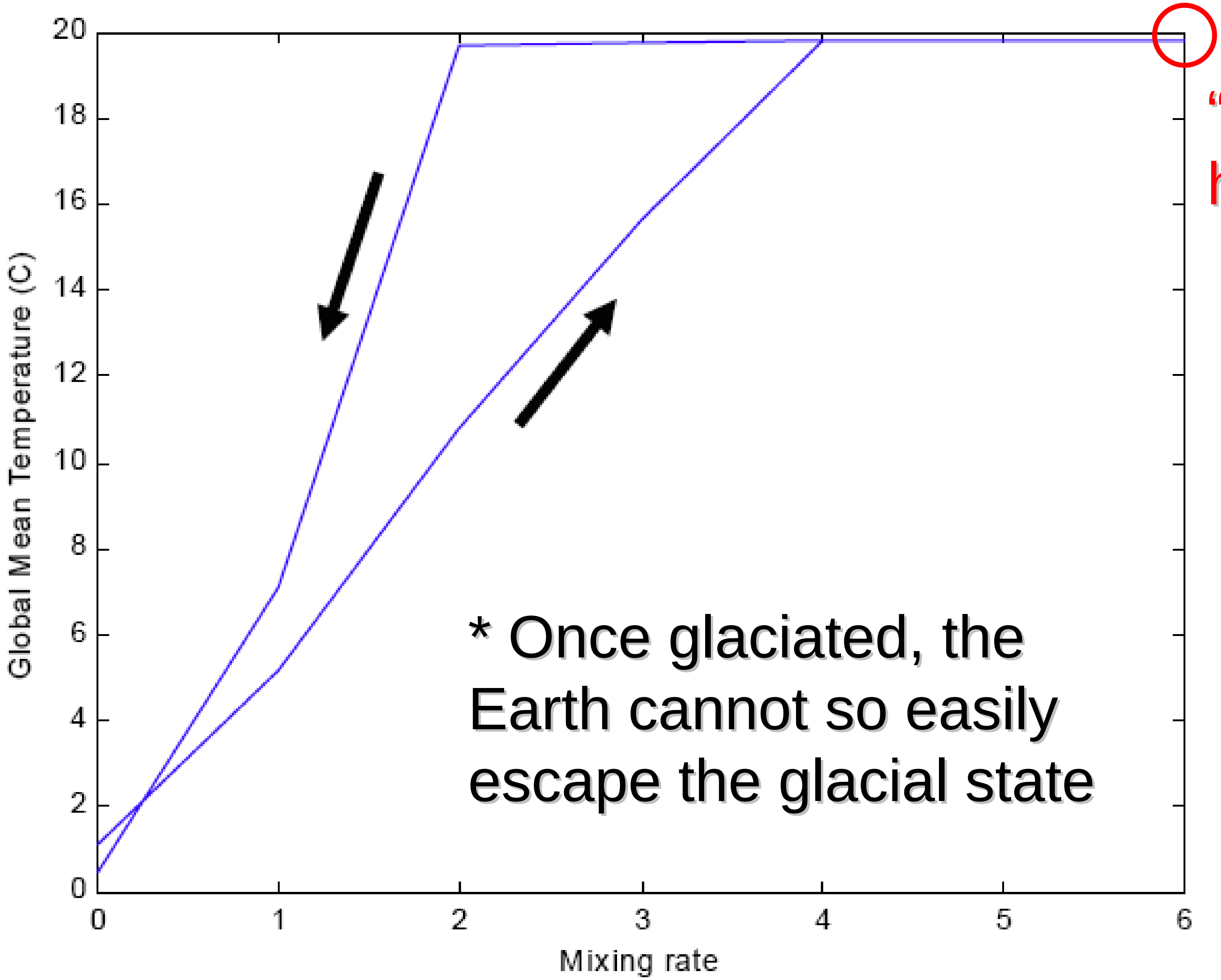
# Global-mean temperature, for each expt (from previous slide) - reveals limited *hysteresis**



“start” here

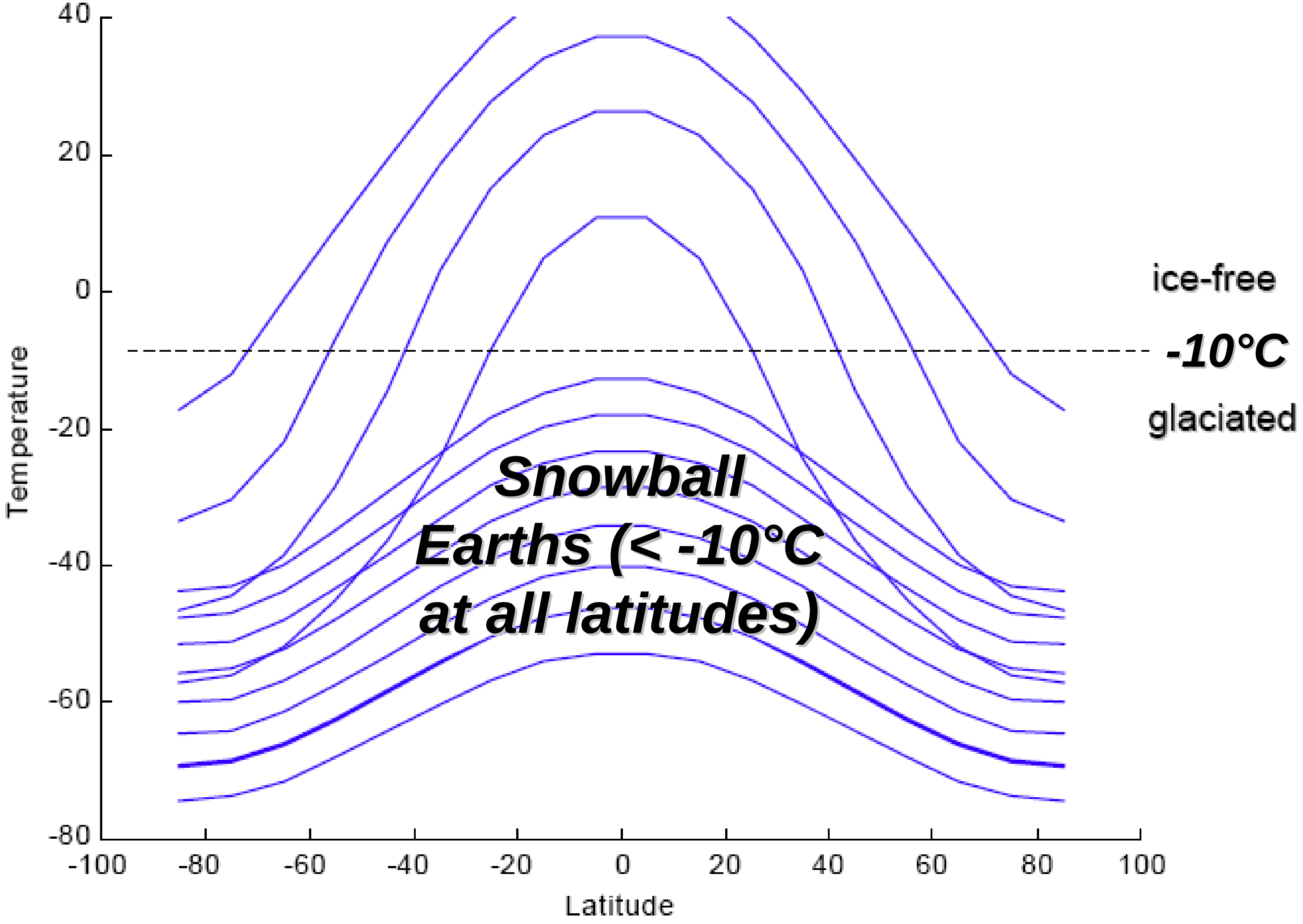
* Once glaciated, the Earth cannot so easily escape the glacial state

Note: expts proceeded with Mixing rate = 6,5,4,3,2,1,0,1,2,3,4 x $10^4$

John Shepherd, University of Southampton

# 1-D EBM : with Ice-Albedo effect ; ***fixed*** mixing 2 x $10^4$ $m^2s^{-1}$ ; ***varying solar constant***

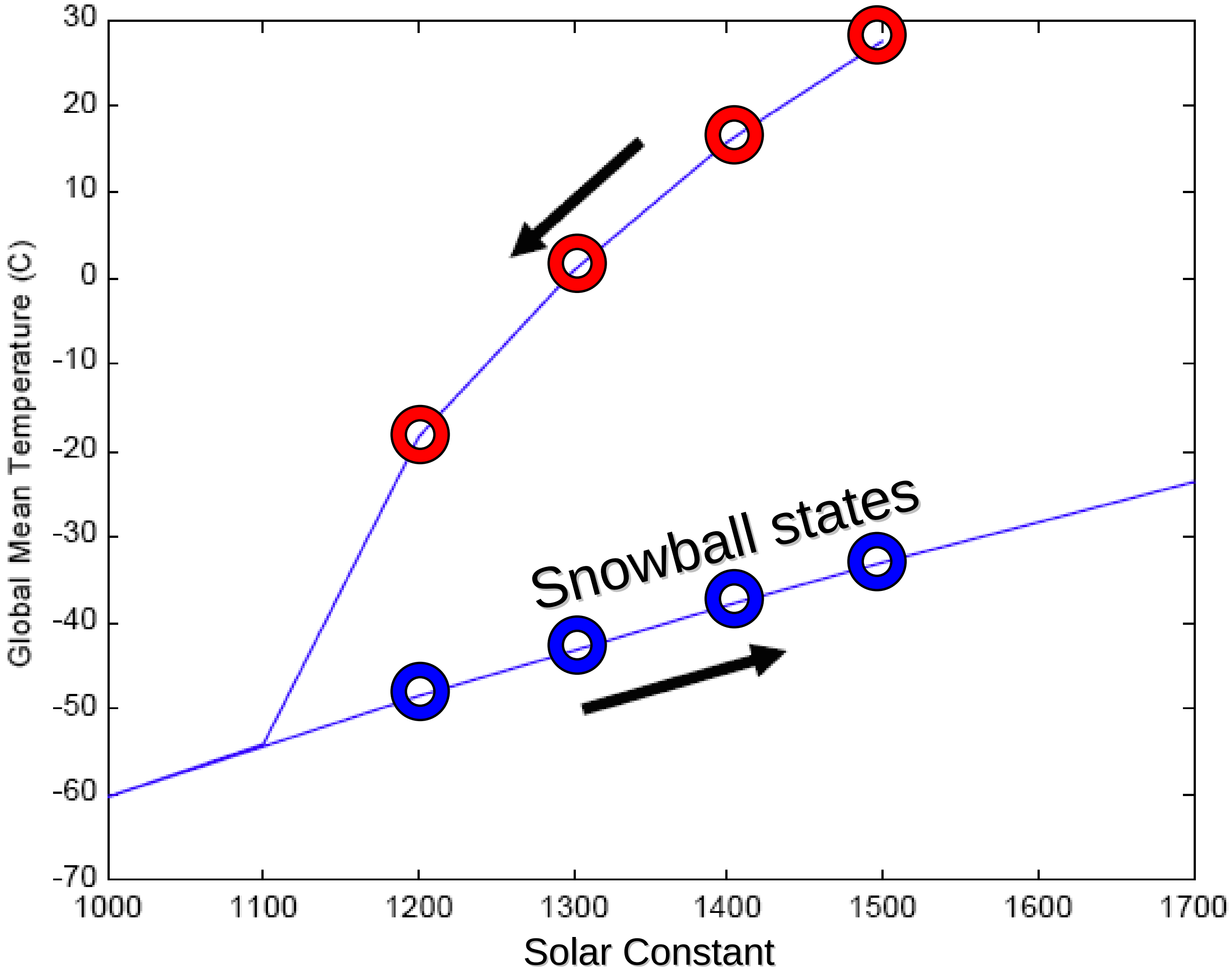
(decreasing, then increasing, in increments of 100 $Wm^{-2}$)



***Snowball Earths (< -10°C at all latitudes)***

John Shepherd, University of Southampton

# Global-mean temperature, for each expt (from previous slide) - reveals ***more extensive hysteresis***



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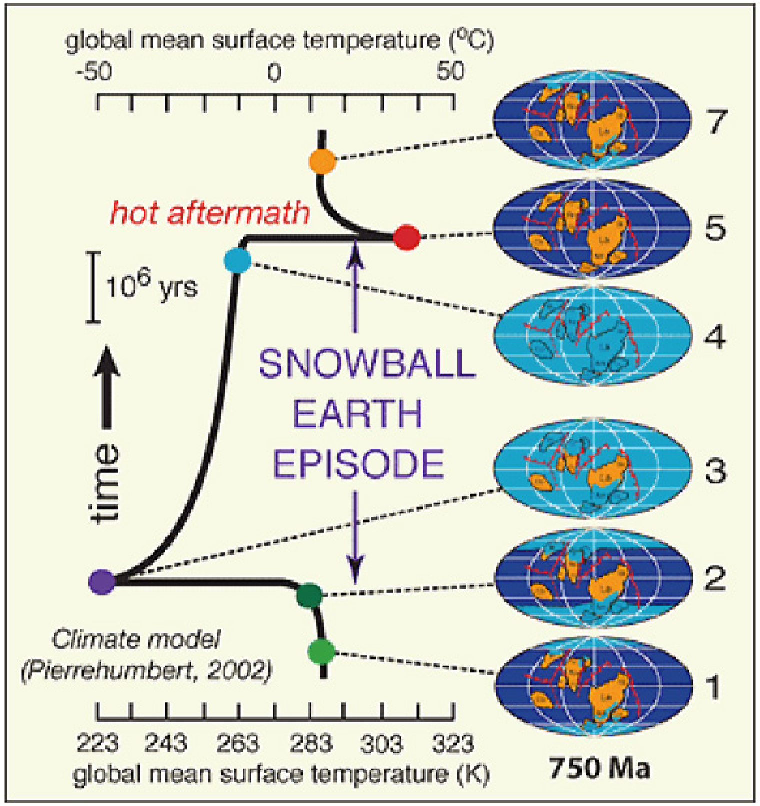
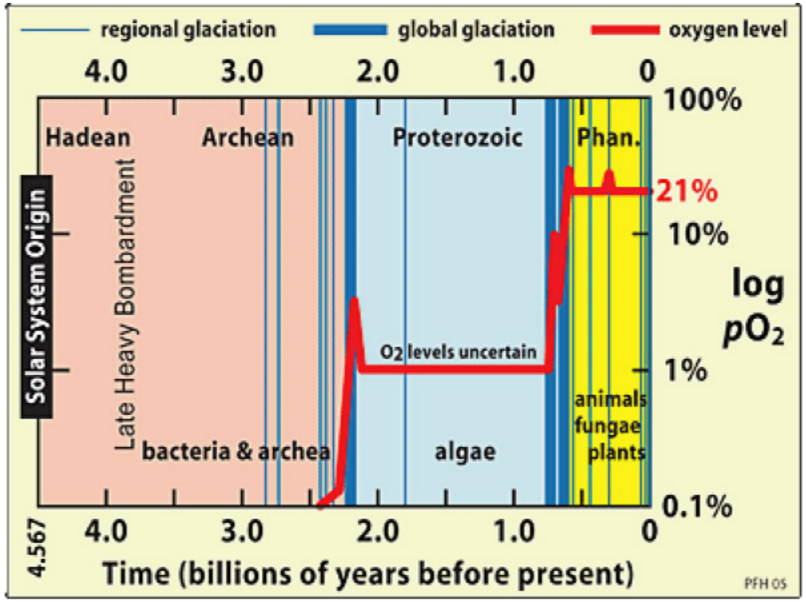
# 4) Snowball Earth Episodes (explained)





http://www.snowballearth.org/images/snwbltvst.gif

***Once the Earth became glaciated, the higher ice albedo reflected more ISWR, prolonging Snowball***

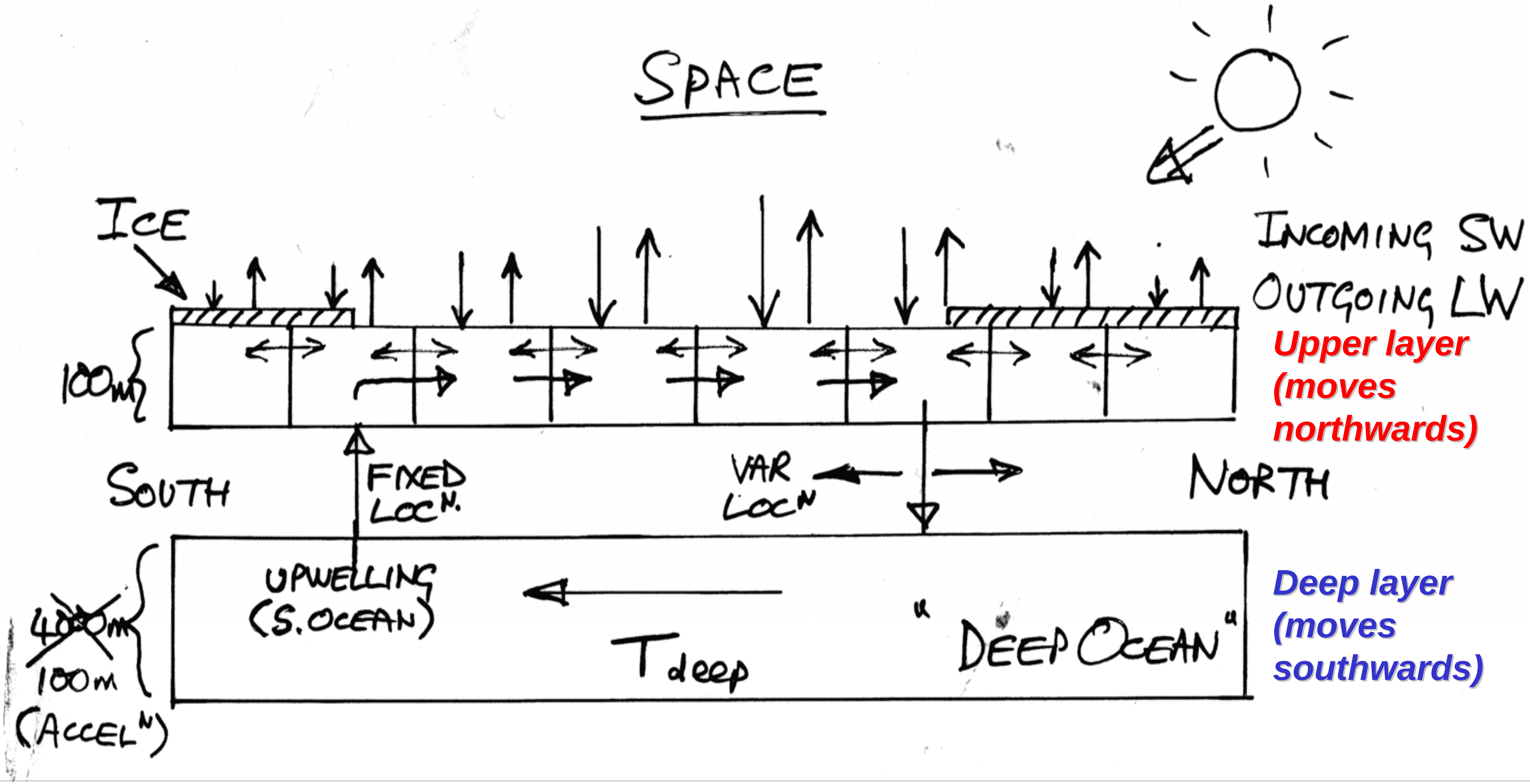
Diagrams courtesy of Snowball Earth (Paul F Hoffmann).

# Adding advective ocean transport

- ❑ Assume same temperature for atmosphere & surface ocean (tight coupling = rapid exchange)
- ❑ Diffusive treatment for mixing (in both)
  - actual location is irrelevant
- ❑ Add advective transport (overturning) in ocean
  - can introduce ***asymmetry*** between N & S hemispheres
  - may be modelled (THC) or specified (stream function)
  - first approximation:

    additional single box for deep ocean …

# 1-D meridional EBM with specified oceanic MOC (THC) & inter-hemispheric asymmetry of heat flux



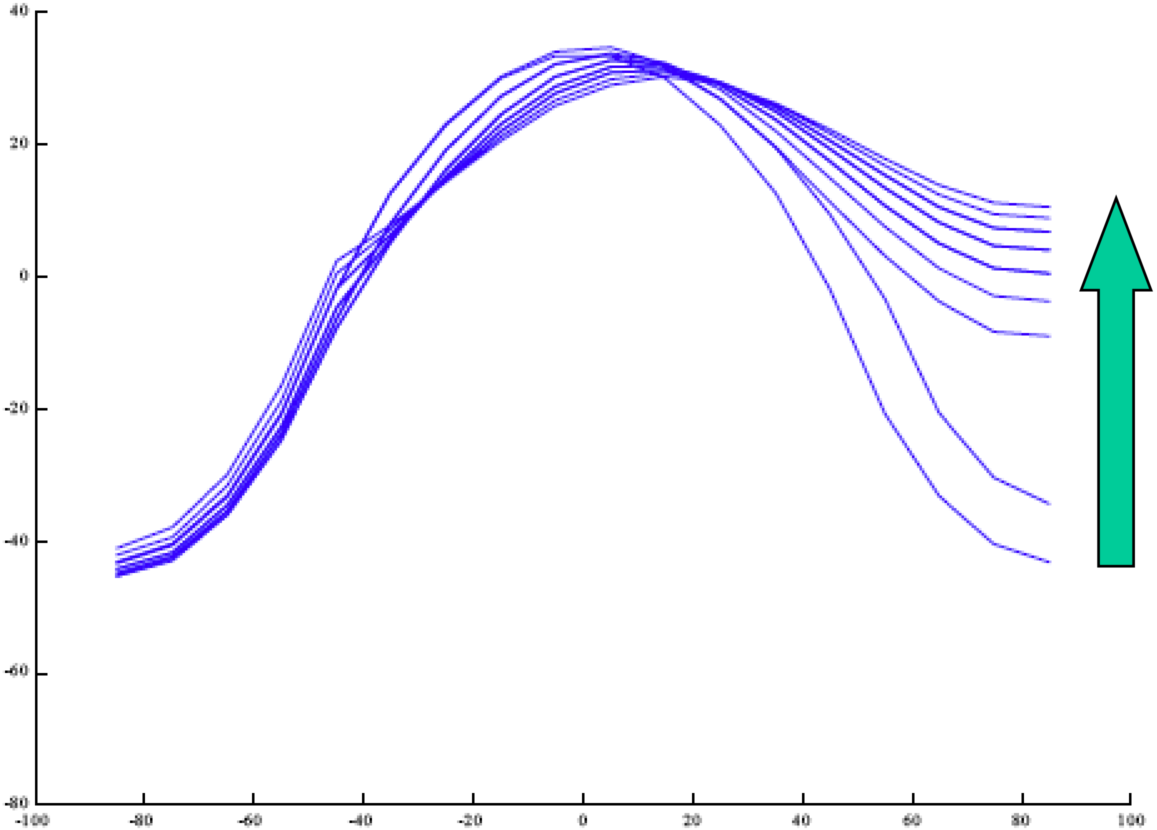
**Upper layer (moves northwards)**

**Deep layer (moves southwards)**

(Sketch courtesy John Shepherd)

N.B. upwelling & downwelling (“sinking”) link surface & deep ocean

# Meridional temperature distribution for ***varying THC rates*** (0 to 60 Sv and back down) : with ice-albedo & water vapour feedbacks, K = 2 x $10^4$ $m^2/s$



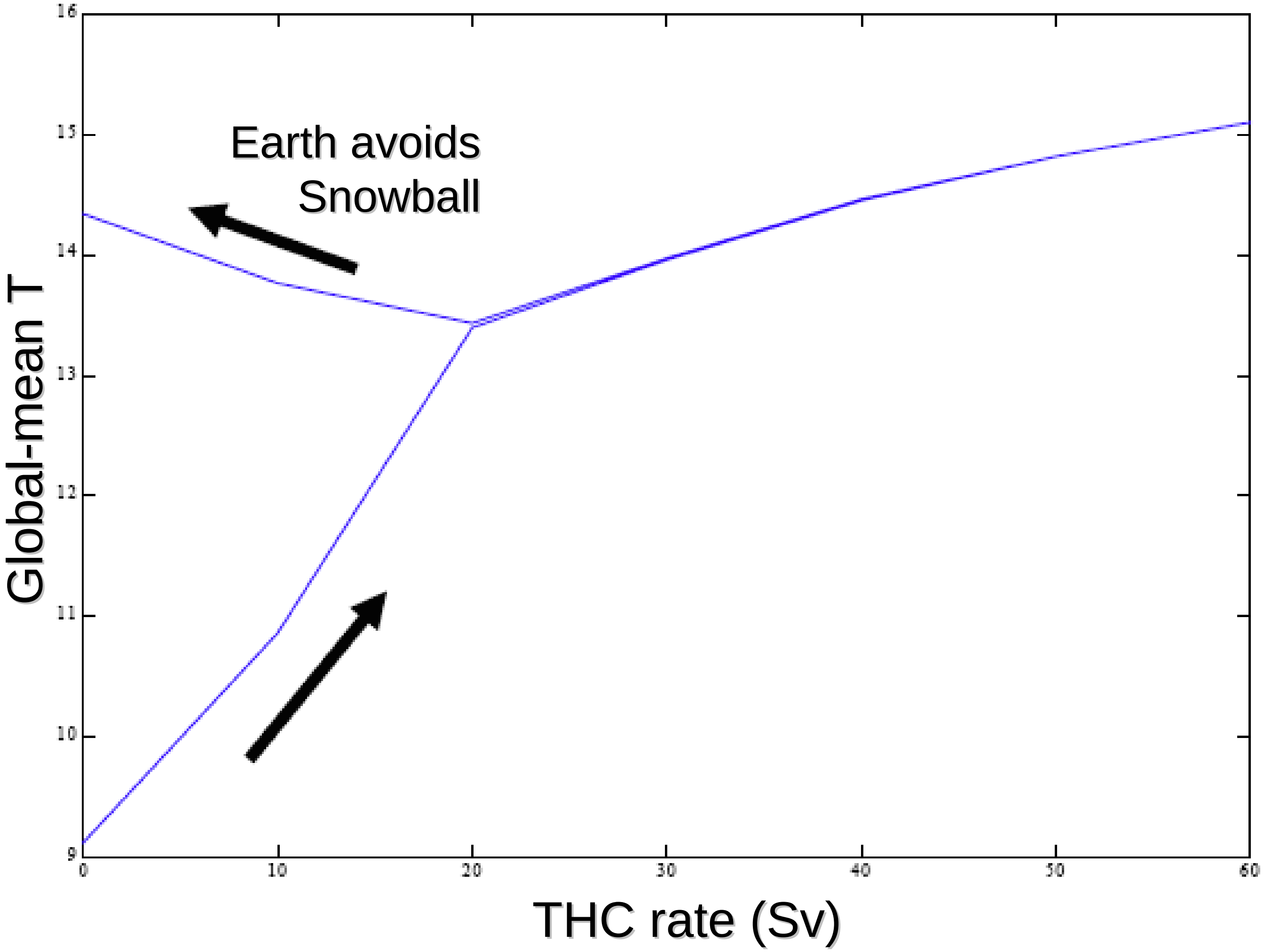
NH high latitudes warm as northward ocean heat transport increases

- Northward surface flow, Southward deep flow (Northward heat transport)
- Northern Hemisphere becomes (and remains) ice-free

John Shepherd, University of Southampton

# Global mean temperature for varying THC rates

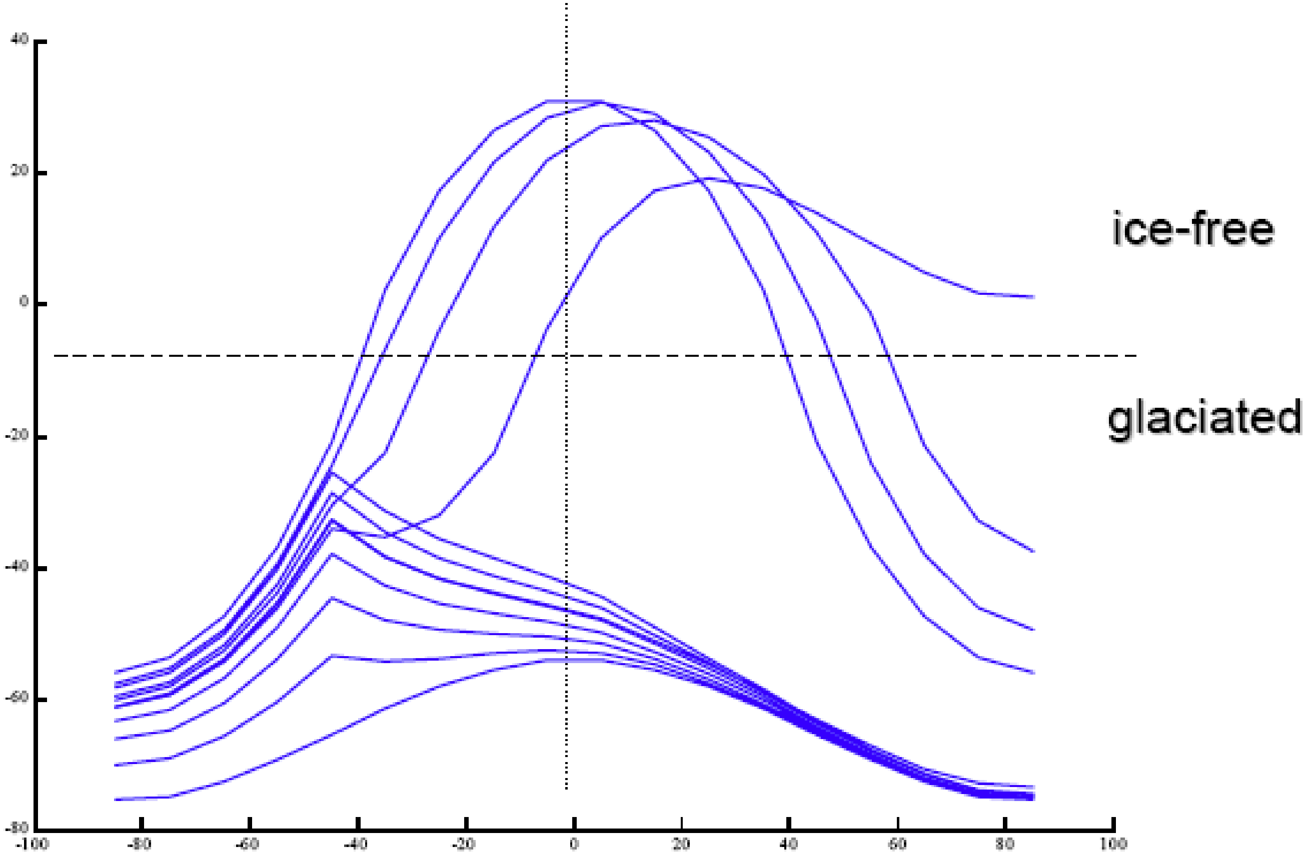
(from previous slide)



Note: expts proceeded with THC rate = 0, 10, … 50, 60, 50, … 10, 0 Sv

# Meridional temperature distribution for ***varying THC rates***

(0 to 60 Sv and back down) : with ice-albedo and water vapour feedbacks,
K = 2 x $10^4$ $m^2/s$ ; ***Ice albedo = 0.75 (increased from 0.7)***

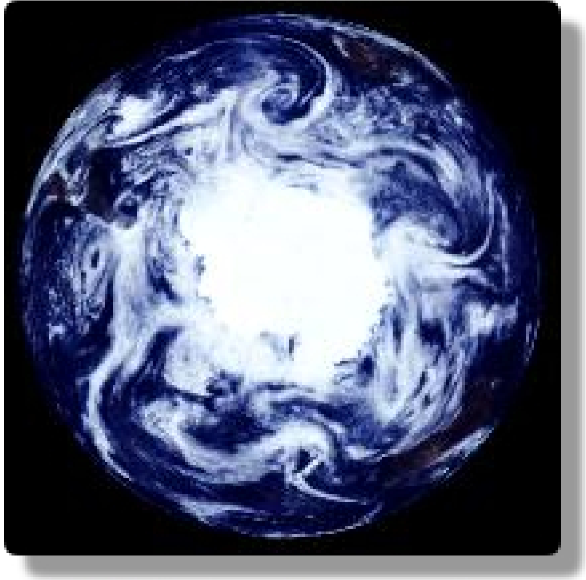


- Southern Hemisphere glaciation spreads to the whole globe: Snowball state
- Harder to escape Snowball state, but possible, with strong enough THC

John Shepherd, University of Southampton

# 5) Meridional heat transport in the Ocean & Atmosphere

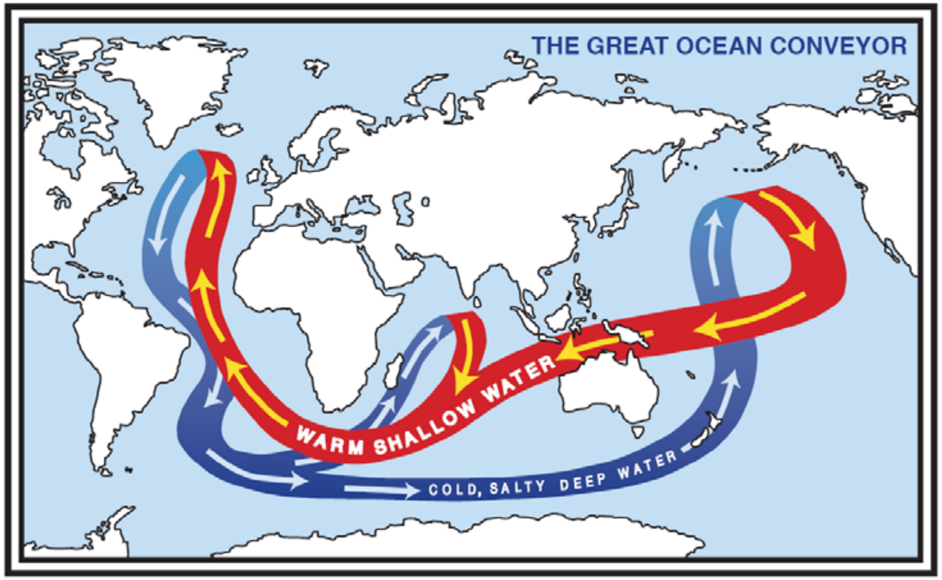
Courtesy of Center for Multiscale Modelling of Atmospheric Processes



Mid-latitude cyclones as viewed from space above the South Pole.

(http://www.cmmap.org/images/learn/climate/ferrel.jpg)

Courtesy of LDEO



The Global Ocean Conveyor Belt

***Atmosphere mixes heat, similarly in both hemispheres; advection more dominant mode of ocean heat transport, typically asymmetric about the Equator, perhaps influencing Snowball state***

# Climate sensitivity re-visited

- Values of $\lambda$ (°K per W m$^{-2}$):
  (e.g. for varying solar constant)

| | | |
|---|---|---|
| black body (S-B law) | 0.30 | |
| + water vapour (Budyko) | 0.46 | |
| **+ ice-albedo feedback** | **0.64** | **(polar ice only)** |
| **if snowball state** | **0.24** | **(more ISWR reflects)** |

# Summary (1)

- Introducing latitudinal effects
  - Net gain (loss) of heat at low (high) latitudes (depending on angle of obliquity)
  - Equator-Pole temperature gradient - net heating/cooling, moderated by meridional heat transport
- 1-D (meridional) EBMs
  - Extension of 0-D EBMs (a connected set of such models)
  - Prescribing insolation as a function of latitude
  - Predicting ***local*** (rather than global) temperature
  - Allowing meridional (temperature) variation in albedo
  - Allowing meridional exchanges of heat

# Summary (2)

- Introducing a heat balance with more terms
  - Steady State balance of ISWR, OLWR, heat transport
  - Parameterize heat transport with Newtonian relaxation (simple) or Fickian diffusion (more appropriate)
- 1-D EBM predictions of temperature vs.latitude
  - With/Without Ice-Albedo feedback
  - With/Without Heat Mixing (and varying strength thereof)
- Prediction of ***multiple equilibria***
  - Bistability - two stable states - within a range of latitude
  - The extent of which is controlled by mixing (stronger mixing = reduced Equator-pole temperature gradients, narrower or eliminated zone of bistability)

# Summary (3)

- 1-D EBM experiments with Fickian diffusion further reveal “Snowball Earth” state
    - Under sufficiently small solar constant
    - And reduced mixing
    - Once in the Snowball state, can’t escape (hysteresis - incrementally increase & decrease control parameter)
    - Also under variable ice albedo (0.70 to 0.75)
- Add explicit ocean heat transport (advective)
    - Preferentially transporting heat northwards (at present)
    - Raising temperature in high northern latitudes
    - Alternative Snowball state also evident under varying ocean heat transport

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