

SOES6006 – Climate Dynamics Practical 1: Stability

The purpose of this practical is to introduce you to the following concepts:

- System equilibria
- Positive and negative feedbacks
- System stability
- Hysteresis
- Chaos

By the end of the practical, you should feel comfortable with these concepts. If you are not, then *tell me*. We will introduce the mathematics underlying the concepts over the next few days, but for now you don't need to worry about this.

In the practical we will be using the MATLAB software package (Available from <http://www.mathworks.com>) Matlab will be used at a less superficial level in the second week of the course.

GETTING STARTED

Download all of the Matlab scripts from the folder entitled 'Matlab Scripts P1 stability' (they end .m) that you find in the folder.

Startup Matlab. Using the "cd" command, navigate to the directory where you downloaded the Matlab scripts.

PART A – A ball on an uneven slope

We begin with an example with little relevance to climate, but it is the best way to start thinking about system stability, because visual representation is consistent with what you would actually observe in the real world. The example is a ball on an uneven slope:

Experiment 1

In the Matlab workspace, type "ball_exp". You will be asked to choose an Experiment. Choose "1". You will be asked to select an initial position for the ball. Accept the default.

A figure will come up showing a section-view of a bowl (steep sides) with a hill slightly off-centre, giving depressions on either side of different depths. A ball will appear with a starting position at rest on one of the slopes.

Before pressing "s" as instructed to start the experiment, guess where you think the ball will eventually come to rest. Then press "s".

The point where the ball comes to rest is one equilibrium point in the system. Having reached equilibrium, the system (in this case the position and velocity of the ball) is not changing in time. "Equilibrium" here is used differently to its use in the laws of motion, where the ball would be in equilibrium if its velocity did not change, even if that velocity were non-zero.

How many equilibrium points are there in the system? That is, how many positions are there where the ball could (in principal) be placed at rest and not start to move? (To answer this question, you need to know that the surface is frictionless, though a ball in motion experiences air resistance.)

To verify this, repeat the experiment, but changing the initial position of the ball. (You may think you don't need to do this, but change your mind after running Experiment 2!) Try to start the ball in different positions until you can get it to end up in all the different equilibria. But don't try too hard....

As you probably guessed, it is easy enough to make the ball end up in two of the equilibria (the two depressions), but almost impossible for it to end up at the third (on top of the off-centre hill). This is because the two depressions are **stable equilibria** and the hill is an **unstable equilibrium**. Understanding this in terms of the ball on a slope is easy, but you need to understand it in terms relevant to systems in general – that is in terms of positive and negative feedbacks:

Do not think of the vertical position of the ball, just its horizontal position. A ball at equilibrium in a depression that is perturbed to the right will experience a force pushing it to the left (this comes from the reactive force from the slope, not from gravity). A ball perturbed to the left will experience a force pushing it to the right. That is, the overall feedback in the system acts in the opposite direction to the perturbation - this is a **negative feedback**. A ball at equilibrium on the hill that is perturbed to the right will experience a force pushing it further to the right (and if it is perturbed to the left, it will experience a force pushing it further to the left). The overall feedback in the system acts in the same direction to the perturbation – this is a **positive feedback**.

A requirement for a system equilibrium to be stable is that negative feedbacks outweigh positive feedbacks.

Experiment 2

In the Matlab workspace, type “ball_exp”. You will be asked to choose an Experiment. Choose “2”. Don’t press “s” to start yet.

The same figure will come up, but instead of having 1 ball, it will have many all along the slope. When you press “s”, the balls will not interact with one another – what you will see is a number of iterations of the experiment plotted together. In effect, you will be running Experiment 1 from many initial positions at the same time.

Another thing to notice is the grey circles at the top of the screen. These indicate the current position of each ball (referencing the colour of the circle below them). To begin with, all the light grey circles are on the left and the dark grey circles on the right. This does not last.

Press “s” to start and let the experiment run until it is obvious where all the balls will finish. What is the relationship between starting position and finishing position?

The most important thing to get out of this is that, when a system has **multiple stable equilibria**, the equilibrium that is reached is a function (and not necessarily a simple one) of the **initial state** or more generally on **past conditions**. That means that knowing the equations governing the behaviour of the system is not sufficient to make predictions. The term **hysteresis** is used to describe such systems, which are also said to have a “memory”.

Experiment 3

In the Matlab workspace, type “ball_exp”. You will be asked to choose an Experiment. Choose “3”. Don’t press “s” to start yet.

The figure will, at first, look identical to the figure in Experiment 2. The difference is in the “forcing” plot on the right. Previous experiments did not experience variability in the external forcing: forces due to gravity, reaction with the surface and air-resistance did not vary except in response to the balls own movements. In this experiment, the balls will be periodically forced to the left and right. (It doesn’t really matter how, but lets say they are magnetic and subject to an oscillating magnetic field.) The key point to note is that **all of the balls are subject to the same forcing; the only difference is the initial state**.

Press “s” and watch the experiment run. You may start to feel sea-sick(!), but let it run to the end (you can tell when this will be from the “forcing” figure). As it runs, try to spot the relationship between the initial position and the final position.

Each of the balls has been subject to the same forcing, with no random component. The position of the ball at the end is therefore a deterministic function of the initial state. Yet, it's hard to see much relationship between the initial and final states. This is because a small perturbation in the initial state leads to a large perturbation in the final state throughout the domain of initial states.

Why is the final state so sensitive to small changes in the initial state?

There are many opportunities for **bifurcation** in the system. Every time a ball approaches the top of the hill at low speed, a small change in its speed (due to a small change in its initial position), can be the difference between making it over the hill or not. Sooner or later, even initially closely spaced balls will fall on either side of the hill (on either side of the **bifurcation**).

This is an extremely simple example of **chaos**. The system appears to be deterministic, because if the initial state is known, the future evolution of the system can be predicted by its governing equations. In practice, however, the initial state needs to be known to infinite precision (or in this case, to the numerical precision of the machine and software) in order to be of use in predicting future states of the system beyond a reasonably short time window.

Chaos is the reason why it is thought that accurate weather forecasting beyond a few weeks will never be possible.

PART B – Radiation balance of a body in space

This example is conceptually a little harder, but mathematically much simpler, and is very relevant to the climate system indeed. We are considering energy (heat) input and output from a body in space, such as the Earth. This will be covered in detail over the coming days, for now, we simply state that the body receives energy at rate $E_{in} = S$ (where S is a constant), emits energy at rate $E_{out} = K\varepsilon T^4$ (where K is a constant, ε is the “emissivity” of the body, and T is its temperature), and that an increase of energy in the system leads to an increase in temperature T .

Experiment 1

In the Matlab workspace, type “radbal_exp”. Accept the default value for S . Enter an emissivity of 1.

A plot will appear, showing the incoming and outgoing energy as a function of temperature. The incoming energy does not vary with temperature, but the outgoing energy does.

(a) Use the plot to determine the equilibrium temperature.

(b) By examining the incoming and outgoing radiation lines, consider what would happen if the temperature was perturbed from this equilibrium (either cooler or warmer). Consider your answer in terms of positive and negative feedbacks, and decide whether you have found a positive or negative equilibrium.

(c) What would the equilibrium temperature be if increased S by 10%? Run the experiment with 110% the default value of S to check your answer if you like.

Experiment 2

Bodies have an emissivity less than 1 if they do not radiate energy like a “black body”. For example, the Earth’s emissivity is reduced by clouds and the greenhouse effect (about which you will learn more over the coming days). Let us suppose that the emissivity reduces as the temperature increases, which is true for Earth because of extra water vapour in the atmosphere as temperature rises. (You will also learn more about this; the function used here to describe this dependence should not be used as an approximation to this effect in the Earth’s atmosphere, however.)

In the Matlab workspace, type “radbal_exp”. Accept the default value for S. Leave emissivity blank. It will then ask you for an added input which is used to determine the rate at which emissivity decreases at higher temperatures. Enter a value of 1.

The lower panel will show how the emissivity varies with temperature, while the upper panel still shows how the radiation terms vary with temperature. The dotted line shows what would have happened if the emissivity were 1.

(a) What effect does this have on the equilibrium of the system? Is the new equilibrium stable or unstable? By examining the effect of the emissivity change on the outgoing radiation line, decide whether this effect is a positive or a negative feedback.

The effect is a positive feedback because an increase in temperature leads to a decrease in emissivity, tending to cause a decrease in outgoing radiation, which further increases temperature). However, the negative feedback due to the outgoing radiation increasing (which reduces the temperature), in response to an increase in temperature is more important. Therefore the new equilibrium is still stable.

(b) What would the equilibrium temperature be if increased S by 10% now? Why is the answer so different from the one you obtained in 1(c) above?

Finally, in the Matlab workspace, type “radbal_exp”. Accept the default value for S. Leave emissivity blank. It will then ask you for an added input which is used to determine the rate at which emissivity decreases at higher temperatures. This time enter a value of 0.7 instead of 1. This will cause the emissivity to decrease more strongly with temperature than before.

You should now see two system equilibria. Concentrate on the equilibrium with the higher temperature.

(c) Determine whether the system is stable or unstable at this equilibrium?

You may have heard the phrase “runaway greenhouse”. This would come about if positive feedback between temperature and emissivity were strong enough, as it is in this example. Fortunately, though this positive feedback is strong in the real climate system, it is nothing like as strong as it is in this hypothetical example at the temperatures considered. In the example only, the equilibrium is unstable because outgoing energy decreases as temperature increases, causing a further increase in temperature.

Copyright statement

This resource was created by the University of Southampton and released as an open educational resource through the 'C-change in GEES' project exploring the open licensing of climate change and sustainability resources in the Geography, Earth and Environmental Sciences. The C-change in GEES project was funded by HEFCE as part of the JISC/HE Academy UKOER programme and coordinated by the GEES Subject Centre.

This resource is licensed under the terms of the **Attribution-Non-Commercial-Share Alike 2.0 UK: England & Wales** license (<http://creativecommons.org/licenses/by-nc-sa/2.0/uk/>).

However the resource, where specified below, contains other 3rd party materials under their own licenses. The licenses and attributions are outlined below:

- The University of Southampton and the National Oceanography Centre, Southampton and its logos are registered trade marks of the University. The University reserves all rights to these items beyond their inclusion in these CC resources.
- The JISC logo, the C-change logo and the logo of the Higher Education Academy Subject Centre for the Geography, Earth and Environmental Sciences are licensed under the terms of the Creative Commons Attribution -non-commercial-No Derivative Works 2.0 UK England & Wales license. All reproductions must comply with the terms of that license.

