QUESTION

(a) Describe briefly some features of a computer implementation of the network simplex method that would improve the efficiency of a standard version of the algorithm.

(b) Show that the following linear programming problem can be formulated as a minimum cost network flow problem.

$$\begin{array}{ll} \text{Minimize} & z = 8x_1 + 7x_2 + 2x_3 + 6x_4 + 2x_5 + 6x_6 \\ & +5x_7 + 8x_8 + 7x_9 + 9x_{10} + 8x_{11} \\ \text{subject to} & x_1, \ldots, x_{11} \geq 0 \\ & x_1 + x_2 + x_3 = 20 \\ & x_3 + x_4 = 16 \\ & x_4 + x_5 = 25 \\ & x_6 + x_7 + x_8 = 10 \\ & x_8 + x_9 + x_{10} = 30 \\ & x_{10} + x_{11} = 32 \\ & x_1 + x_6 \leq 23 \end{array}$$

Starting with a solution in which x_2, x_4, x_5, x_7, x_9 and x_{11} take positive values, and the last constraint is satisfied as a strict inequality, use the network simplex method to solve the problem.

ANSWER

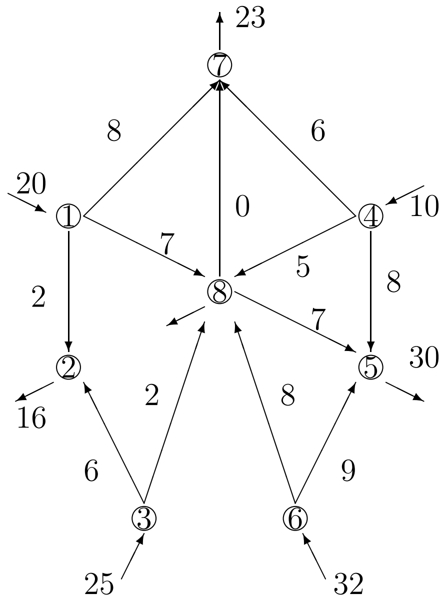
(a) The main points are:

- compute dual variables by updating their values from the previous iteration, rather than performing a complete recalculation
- compute reduced costs for a subset of arcs, and if there are negative reduced costs, then choose the entering arc from this subset.

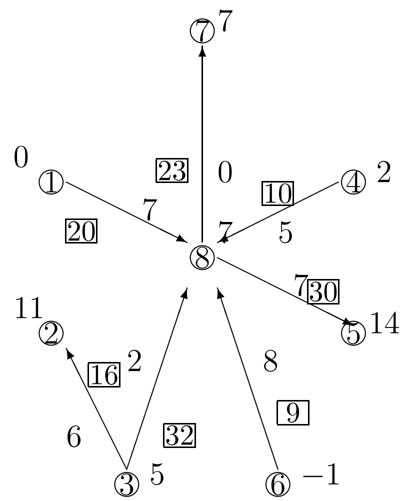
(b) The constraints can be written as

$$\begin{array}{rcl} x_1 + x_2 + x_3 & = & 20 \\ -x_3 - x_4 & = & -16 \\ x_4 + x_5 & = & 25 \\ x_6 + x_7 + x_8 & = & 10 \\ -x_8 - x_9 + x_{10} & = & -30 \\ x_{10} + x_{11} & = & 32 \\ -x_1 - x_6 - s & = & -23 \\ -x_2 - x_5 - x_7 + x_9 - x_{11} + s & = & -18 \end{array}$$

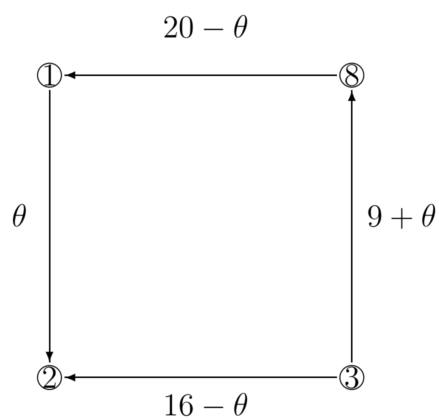
where s is a slack variable, and the last redundant constraint is deduced from the others.



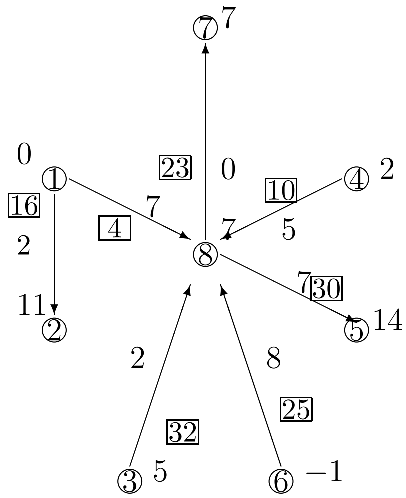
Initial tree solution



Non basic	$y_i + c_{ij} - y_j$	
(1,2)	-9	(Entering arc (1,2))
(1,7)	1	
(4,5)	1	
(4,7)	-4	
(6,5)	-6	

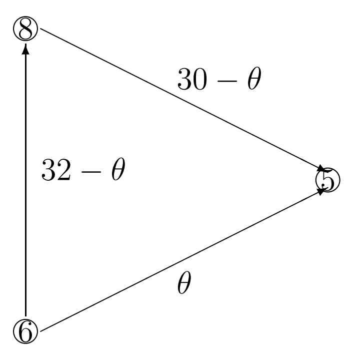


$\theta = 16$ leaving arc (3,2)

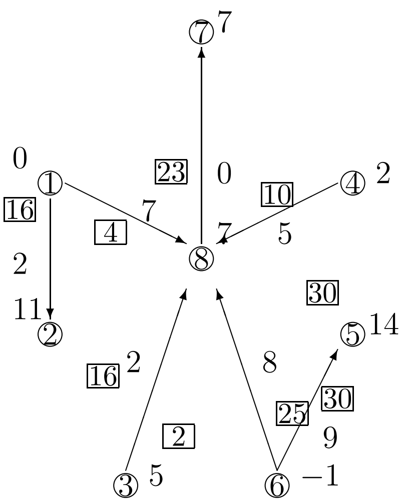


Non-basic	$y_i + c_{ij} - y_j$
(1,7)	1
(3,2)	9
(4,5)	−4
(4,7)	1
(6,5)	−6

Entering arc (6,5)



Leaving arc (8,5)



Non-basic	$y_i + c_{ij} - y)j$
(1,7)	1
(3,2)	9
(4,5)	2
(4,7)	1
(8,5)	6

Thus we have an optimal solution.

$x_3 = 16$ $x_2 = 4$ $x_5 = 25$ $x_{11} = 2$ $x_{10} = 30$ $x_7 = 10$ $x_1 = x_4 = x_6 = x_8 = x_9 = 0$ $z = 446$