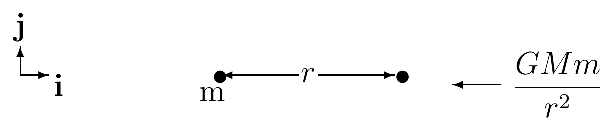
Question

A particle of mass m moves in a straight line form a fixed particle of mass M under their influence of their mutual gravitational attraction. Show that the potential of the gravitational force is $-\dfrac{Gmm}{r}$, where R is the distance between two masses. A rocket departs from the earth in a straight line. Shortly after lift-off when the engines have stopped firing the rocket has speed u.

(a) Show that in order that the rocket can escape the earth's gravitational field $u > u_E$, where u_E is the so called escape velocity for a particle. (treat the gravitational field of the earth as that of a particle with the mass of the earth and write down conservation of energy for the rocket.)

(b) Calculate u_E given that $G = 6.6726 \times 10^{-11}$ Nm^{2}kg^{-2}, the mass of the earth is 5.976×10^{24}kg and the radius of the earth is 6.38×10^6m.

Answer



Gravitational Force $\mathbf{F} = -\dfrac{GMm}{r^2}\mathbf{i}$

Potential $= -\displaystyle\int F\,dr = -\dfrac{GMm}{r}$

(a) Conservation of energy: Kinetic Energy + Potential Energy = Constant

Therefore $\dfrac{1}{2}mu^2 - \dfrac{GMm}{r} = \text{constant}$

Initially $u = v$, and $r = R$ (the radius of the earth)

Therefore $\dfrac{1}{2}mv^2 - \dfrac{GMm}{r} = \dfrac{1}{2}mv^2 - \dfrac{GMm}{R}$

(b) For a rocket to escape the Earth's gravitational field requires $v \geq 0$ as $r \to \infty$, hence

$$\frac{1}{2}m^2 \geq \frac{GMm}{R} \Rightarrow v \geq \sqrt{\frac{2GM}{R}}$$

Using the Earth's data gives an escape velocity of 11.2kms^{-1}