# Applications of Partial Differentiation
## *Extremes*

**Question**

Find and classify the critical points of the function

$$f(x, y) = \cos x + \cos y$$

**Answer**

$$
\begin{aligned}
f_1 &= -\sin x \\
f_2 &= -\sin y \\
A &= f_{11} = -\cos x \\
B &= f_{12} = 0 \\
C &= f_{22} = -\cos y.
\end{aligned}
$$

The critical points are $(m\pi, n\pi)$ where $m$ and $n$ are integers.

Here $B^2 - AC = -\cos(m\pi)\cos(n\pi) = (-1)^{m+n+1}$ which is negative if $m+n$ is even, and positive if $m + n$ is odd.

$m+n$ even: $\Rightarrow f$ has a saddle point at $(m\pi, n\pi)$. $m+n$ odd and $m$ is odd: $\Rightarrow f$ has local (and absolute) minimum value, $-2$, at $(m\pi, n\pi)$. $m + n$ odd and $m$ is even: $\Rightarrow f$ has a local (and absolute) maximum value, $2$, at $(m\pi, n\pi)$.