Applications of Partial Differentiation
Extremes

Question
Find and classify the critical points of the function

$$f(x, y) = \frac{x}{y} + \frac{8}{x} - y$$

Answer

$$\begin{aligned} f_1(x, y) &= \frac{1}{y} - \frac{8}{x^2} = 0 \quad \text{if } 8\text{y} = \text{x}^2 \\ f_2(x, y) &= -\frac{x}{y^2} - 1 = 0 \quad \text{if } \text{x} = -\text{y}^2 \end{aligned}$$

For critical points: $8y = x^2 = y^4$, so $y = 0$ or $y = 2$.
$f(x, y)$ is not defined when $y = 0$, so the only critical point is $(-4, 2)$.
At $(-4, 2)$ we have

$$\begin{aligned} A &= f_{11} = \frac{16}{x^3} = -\frac{1}{4} \\ B &= f_{12} = -\frac{1}{y^2} = -\frac{1}{4} \\ C &= f_{22} = \frac{2x}{y^3} = -1 \end{aligned}$$

Thus $B^2 - AC = \frac{1}{16} - \frac{1}{4} < 0$, and $(-4, 2)$ is a local maximum.