Question

Find the solution of the following differential equations with the given initial condition.

1. $\dfrac{dy}{dx} = 8x^3e^{-2y} \qquad y(1) = 0 \qquad (*)$
2. $\dfrac{dy}{dx} = y\sin x \qquad y(\pi) = -3$
3. $\dfrac{dy}{dx} = x^2(1+y) \qquad y(0) = 3 \qquad (*)$

Answer

a) $\dfrac{dy}{dx} = 8x^3e^{-2y} \Rightarrow \displaystyle\int e^{2y}dy = \int 8x^3dx \Rightarrow \frac{1}{2}e^{2y} = 2x^4 + c$

now, when $x = 1, y = 0$ so

$\dfrac{1}{2}e^{2(0)} = 2(1)^4 + c \Rightarrow \dfrac{1}{2} = 2 + c \Rightarrow c = \dfrac{-3}{2}$

hence $\frac{1}{2}e^{2y} = 2x^4 - \frac{3}{2} \Rightarrow y = \frac{1}{2}\ln(4x^3 - 3)$

b) $\dfrac{dy}{dx} = y\sin x \Rightarrow \displaystyle\int \frac{1}{y}dy = \int \sin x dx \Rightarrow \ln y = -\cos x + c$

$\Rightarrow y = ke^{-\cos x}$

when $x = \pi, y = -3$ so choose k so that

$-3 = ke^{-\cos(\pi)} \Rightarrow -3e^{-1} = k \Rightarrow y = -3e^{(-\cos x - 1)}$

c) $\dfrac{dy}{dx} = x^2(1+y) \Rightarrow \displaystyle\int \frac{dy}{1+y} = \int x^2dx \Rightarrow \ln(1+y) = \frac{1}{3}x^3 + c$

$1 + y = e^{\frac{1}{3}x^3 + c} \Rightarrow y = ke^{\frac{1}{3}x^3} - 1$

when $x = 0, y = 3 \Rightarrow 3 = ke^{\frac{1}{3}(0)^3} - 1 \Rightarrow k = 4$ hence $y = 4e^{\frac{1}{3}x^3} - 1$