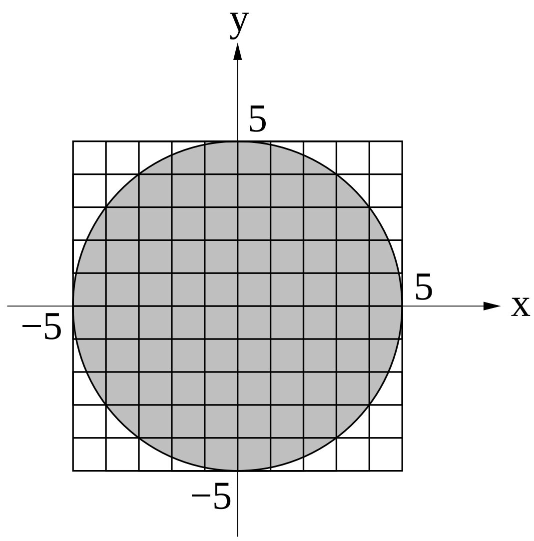
Multiple Integration
Double Integrals

Question

D is the disk $x^2 + y^2 \leq 25$

P is the partition of the square $-5 \leq x \leq 5$, $-5 \leq y \leq 5$ into one hundred squares of dimensions 1×1, shown below



$$J = \int\int_D f(x, y)\, dA$$

where $f(x, y) = 1$.

Approximate J by calculating the Riemann sums $R(f, P)$ with the given points (x^*_{ij}, y^*_{ij}) in the small squares. Use of symmetry will speed things up.

(a) (x^*_{ij}, y^*_{ij}) is the corner of each square closest to the origin.

(b) (x^*_{ij}, y^*_{ij}) is the corner of each square farthest from the origin.

(c) (x^*_{ij}, y^*_{ij}) is the centre of each square.

(d) Evaluate J

(e) Repeat 2(c), replacing $f(x, y) = 1$ with $f(x, y) = x^2 + y^2$.

Answer

$$J = \int\int_D 1\, dA$$

(a) $R = 4 \times 1 \times [5+5+5+5+4] = 96$

(b) $R = 4 \times 1 \times [4+4+4+3+0] = 60$

(c) $R = 4 \times 1 \times [5+5+4+4+2] = 80$

(d) $J =$area of disk$= \pi(5^2) \approx 78.54$

(e) $f(x,y) = x^2 + y^2$.

$$\begin{aligned} R &= 4 \times 1 \times [f(\frac{1}{2},\frac{1}{2}) + f(\frac{3}{2},\frac{1}{2}) + f(\frac{5}{2},\frac{1}{2}) + f(\frac{7}{2},\frac{1}{2}) + f(\frac{9}{2},\frac{1}{2}) \\ &\quad + f(\frac{1}{2},\frac{3}{2}) + f(\frac{3}{2},\frac{3}{2}) + f(\frac{5}{2},\frac{3}{2}) + f(\frac{7}{2},\frac{3}{2}) + f(\frac{9}{2},\frac{3}{2}) \\ &\quad + f(\frac{1}{2},\frac{5}{2}) + f(\frac{3}{2},\frac{5}{2}) + f(\frac{5}{2},\frac{5}{2}) + f(\frac{7}{2},\frac{5}{2}) \\ &\quad f(\frac{1}{2},\frac{7}{2}) + f(\frac{3}{2},\frac{7}{2}) + f(\frac{5}{2},\frac{7}{2}) \\ &\quad + f(\frac{1}{2},\frac{9}{2}) + f(\frac{3}{2},\frac{9}{2})] \\ &= 918 \end{aligned}$$