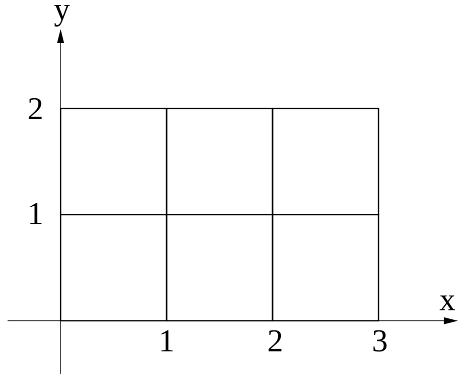
Multiple Integration
Double Integrals

Question

$$I = \int\int_D (5 - x - y)\, dA$$

D is the rectangle $0 \leq x \leq 3$, $0 \leq y \leq 2$.
P is the partition of D into six squares, each of side 1, as shown below.



For the following choices of points (x^*_{ij}, y^*_{ij}), calculate the Riemann sum for I

(a) The upper left corner of each square

(b) The upper right corner of each square

(c) The lower left corner of each square

(d) The lower right corner of each square

(e) The centre of each square

(f) Evaluate I by interpreting it as a volume.

(g) Repeat 1(e), replacing $f(x, y) = 5 - x - y$ with $f(x, y) = e^x$.

Answer

(a) $f(x, y) = 5 - x - y$

$$\begin{aligned} R &= 1 \times [f(0,1) + f(0,2) + f(1,1) + f(1,2) \\ &\quad + f(2,1) + f(2,2)] \\ &= 4 + 3 + 3 + 2 + 2 + 1 = 15 \end{aligned}$$

(b)

$$\begin{aligned} R &= 1 \times [f(1,1) + f(1,2) + f(2,1) + f(2,2) \\ &\quad + f(3,1) + f(3,2)] \\ &= 3 + 2 + 2 + 1 + 1 + 0 = 9 \end{aligned}$$

(c)

$$\begin{aligned} R &= 1 \times [f(0,0) + f(0,1) + f(1,0) + f(1,1) \\ &\quad + f(2,0) + f(2,1)] \\ &= 5 + 4 + 4 + 3 + 3 + 2 = 21 \end{aligned}$$

(d)

$$\begin{aligned} R &= 1 \times [f(1,0) + f(1,1) + f(2,0) + f(2,1) \\ &\quad + f(3,0) + f(3,1)] \\ &= 4 + 3 + 3 + 2 + 2 + 1 = 15 \end{aligned}$$

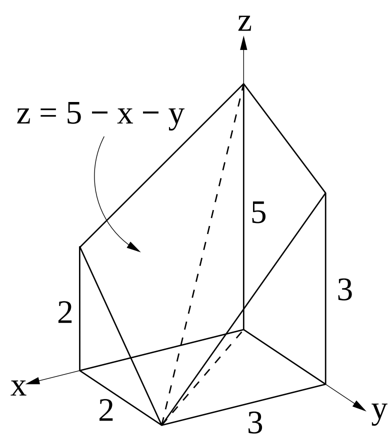
(e)

$$\begin{aligned} R &= 1 \times [f(\frac{1}{2}, \frac{1}{2}) + f(\frac{1}{2}, \frac{3}{2}) + f(\frac{3}{2}, \frac{1}{2}) + f(\frac{3}{2}, \frac{3}{2}) \\ &\quad + f(\frac{5}{2}, \frac{1}{2}) + f(\frac{5}{2}, \frac{3}{2})] \\ &= 4 + 3 + 3 + 2 + 2 + 1 = 15 \end{aligned}$$

(f)

$$I = \int\int_D (5 - x - y)\, dA$$

is the volume of the solid shown below.



The solid is split into two pyramids by the vertical plane through the z-axis and the point $(3, 2, 0)$.

One of these pyramids has a base in the plane $y = 0$, and the other has a base in the plane $z = 0$.

The sum of the volumes of these pyramids gives I.

$$I = \frac{1}{3}\left(\frac{5+2}{2}(3)(2)\right) + \frac{1}{3}\left(\frac{5+3}{2}(2)(3)\right) = 15$$

(g)

$$\begin{aligned} R &= (e^{1/2} + e^{1/2} + e^{3/2} + e^{3/2} + e^{5/2} + e^{5/2}) \\ &\approx 32.63 \end{aligned}$$