Question
In the question YOU MAY ASSUME

(i) that all Calls and Puts are of European type,

(ii) that (whether bought long or short) all Calls may be purchased for a constant value C and all Puts may be purchased for a constant value P,

(iii) that all calculations are to be made from the point of view of the holder of the option (rather than that of the writer).

(a) A STRADDLE is an option strategy that consists of a position where one is a long one Call and long one Put, both with same strike E and expiry T. At expiry the underlying has a value $S(T)$. What conditions must $S(T)$ satisfy in order for a straddle to be profitable?

Draw a profit diagram for a straddle, plotting the profit at expiry against $S(T)$. If an investor buys a straddle, what view is she or he taking of the likely behaviour of the underlying?

(b) A BUTTERFLY SPREAD is an option strategy that consists of a position where one is long one Call with a strike $E - K$, long one call with a strike $E + K$ and short two calls, both with strike E, where the constant K satisfies $4C < K < E$. All Calls are assumed to have the same expiry T. At expiry the underlying has a value $S(T)$. What conditions must $S(T)$ satisfy in order for a butterfly spread to be profitable?

Draw a profit diagram for a butterfly spread, plotting the profit at expiry against $S(T)$. If an investor buys a butterfly spread, what view is she or he taking of the likely behaviour of the underlying?

Answer

(a) The total payoff from a straddle is that due to one call and one put, i.e.

$$\max(\mathrm{S}-\mathrm{E},0)+\max(\mathrm{E}-\mathrm{S},0).$$

In order for the straddle to be profitable to the holder we therefore require

$$\max(\mathrm{S}-\mathrm{E},0)+\max(\mathrm{E}-\mathrm{S},0)-\mathrm{C}-\mathrm{P}>)$$

There are two cases:

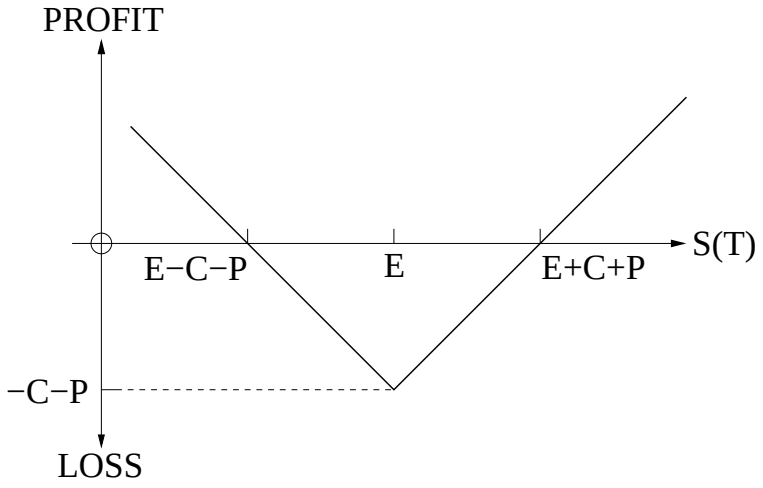
(i) $S > E$ in which case profit= $S_E + 0 - C - P$ and so for profit> 0 we need $S > E + C + P$

(ii) $S < E$ in which case profit= $0+E-S-C-P$ and so for profit> 0 we need $S < E - C - P$.

So for profit we need

$$\text{EITHER S} > \text{E} + \text{C} + \text{P} \quad \text{OR} \quad \text{S} < \text{E} - \text{C} - \text{P}$$

Profit diagram:-



(Diagram must have all these labels for full marks)

-Investor buys a straddle if she or he believes that the underlying will move a long way in price, but doesn't know which way.

(b) Payoff from Butterfly Spread=

$$\max(\mathrm{S}-\mathrm{E}+\mathrm{K},0)+\max(\mathrm{S}-\mathrm{E}-\mathrm{K},0)-2\max(\mathrm{S}-\mathrm{E},0)$$

$\Rightarrow$ Profit $=\max(\mathrm{S}-\mathrm{E}+\mathrm{K},0)+\max(\mathrm{S}-\mathrm{E}-\mathrm{K},0)-2\max(\mathrm{S}-\mathrm{E},0)-4\mathrm{C}$

There are now 4 cases to consider:-

(i) $S < E - K$
Profit $= 0 + 0 - 2(0) - 4\mathrm{C}$
$= -4C$

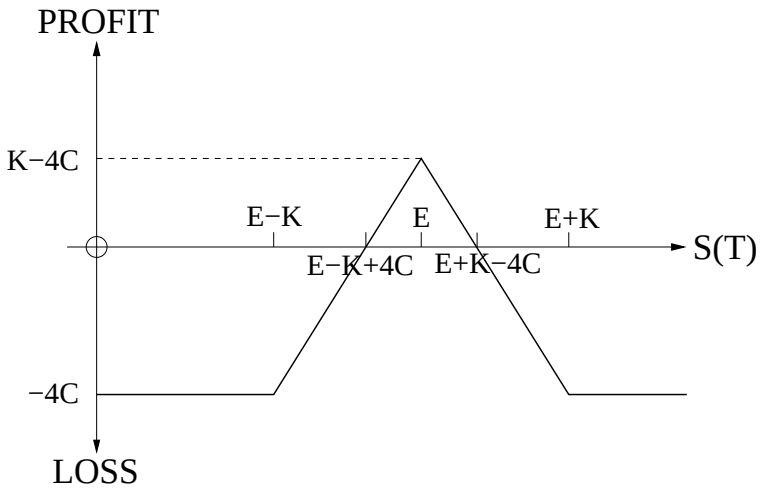
(ii) $E - K < S < E$
Profit $= \mathrm{S} - \mathrm{E} + \mathrm{K} + 0 - 2(0) - 4\mathrm{C}$
$= S - E + K - 4C$

(iii) $E < S < E + K$
Profit $= \mathrm{S} - \mathrm{E} + \mathrm{K} + 0 - 2(\mathrm{S} - \mathrm{E}) - 4\mathrm{C}$
$= -S + E + K - 4C$

(iv) $E + K < S$
Profit $= \mathrm{S} - \mathrm{E} + \mathrm{K} + \mathrm{S} - \mathrm{E} - \mathrm{K} - 2\mathrm{S} + 2\mathrm{E} - 4\mathrm{C}$
$= -4C$

Thus to be profitable we need

$$\begin{aligned} S - E + K - 4C > 0 &\Rightarrow S > 4C + E - K \quad \text{or} \\ -S + E + K - 4C > 0 &\Rightarrow S < -4C + E + K \\ \text{i.e.E} - \mathrm{K} + 4\mathrm{C} < &\ S \ < E + K - 4C \end{aligned}$$



(Again, need all labels for full marks)

An investor buys a butterfly spread if she or he considers that changes in the price of the underlying will be small.