QUESTION

**(a)** Using partial fractions evaluate $\int_1^2 \frac{1}{x(x+2)}\,dx$.

**(b)** A double integral is defined by

$$\int_{y=0}^{y=1}\int_{x=\sqrt{y}}^{x=1} \exp(x^3)\,dx\,dy.$$

**(i)** Sketch the region of integration.

**(ii)** Rewrite the integral so that the integration with respect to $y$ may be performed first, and hence evaluate the integral.

ANSWER

**(a)** $\dfrac{1}{x(x+2)} = \dfrac{A}{x} + \dfrac{B}{x+2} = \dfrac{A(x+2)+Bx}{x(x+2)}$

Therefore $1 = A(x+2) + Bx$.

$x = 0;\ 1 = 2A + 0,\ A = \dfrac{1}{2}$

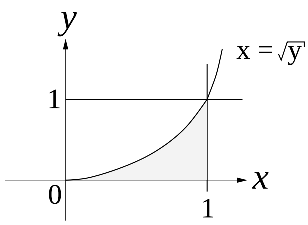
$x = -2;\ 1 = 0 - 2B,\ B = -\dfrac{1}{2}$

Therefore

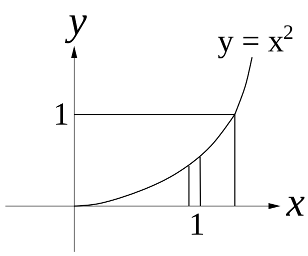
$$\begin{aligned}
\int_1^2 \frac{1}{x(x+2)}\,dx &= \int_1^2 \left\{\frac{\frac{1}{2}}{x} - \frac{\frac{1}{2}}{x+2}\right\} dx \\
&= \left[\frac{1}{2}\ln x - \frac{1}{2}\ln(x+2)\right]_1^2 \\
&= \left(\frac{1}{2}\ln 2 - \frac{1}{2}\ln 4\right) - \left(\frac{1}{2}\ln 1 - \frac{1}{2}\ln 3\right) \\
&= \frac{1}{2}\ln\left(\frac{2\times 3}{4}\right) \\
&= \frac{1}{2}\ln\left(\frac{3}{2}\right)
\end{aligned}$$

**(b)** $\displaystyle\int_{y=0}^{y=1}\int_{x=\sqrt{y}}^{x=1} e^{x^3}\,dx\,dy$

**(i)** $x = \sqrt{y},\ y = x^2,\ x = 1,\ y = 0,\ y = 1;$




**(ii)**




$$\int_{x=0}^{x=1} \int_{y=0}^{y=x^2} e^{x^3}\, dy\, dx$$

$$I = \int_{x=0}^{1} \left[ye^{x^3}\right]_{y=0}^{y=x^2} dx = \int_{x=0}^{1} x^2 e^{x^3}\, dx = \left[\frac{1}{3}e^{x^3}\right]_0^1 = \frac{1}{3}(e-1)$$