QUESTION An area A is enclosed between the curve $y = x^{\frac{1}{3}}$, the x-axis and the lines $x = 0$ and $x = 8$.

(i) Find the magnitude of A.

(ii) Calculate the coordinates of the centroid of A.

(iii) Find the volume generated when A is rotated about the y-axis.

ANSWER
DIAGRAM

(i) $A = \int_0^8 y\,dx = \int_0^8 x^{\frac{1}{3}}\,dx = \left[\frac{x^{\frac{4}{3}}}{\frac{4}{3}}\right]_0^8 = \frac{3}{4}(8^{\frac{4}{3}} - 0) = \frac{3}{4}(2^4) = 12$

(ii) The coordinates of the centroid are $(\overline{x}, \overline{y})$.

$$A\overline{x} = \int_0^8 xy\,dx = \int_0^8 x^{\frac{4}{3}}\,dx = \left[\frac{x^{\frac{7}{3}}}{\frac{7}{3}}\right]_0^8 = \frac{3}{7}(8^{\frac{7}{3}}) = \frac{3}{7}(2^7)$$

therefore $\overline{x} = \frac{3(128)}{7(12)} = \frac{32}{7}$

$$A\overline{y} = \int_0^8 \frac{1}{2}y^2\,dx = \int_0^8 \frac{1}{2}x^{\frac{2}{3}}\,dx = \frac{1}{2}\left[\frac{x^{\frac{5}{3}}}{\frac{5}{3}}\right]_0^8$$

i.e. $A\overline{y} = \frac{3}{10}(8^{\frac{5}{3}}) = \frac{3(32)}{10}$

Therefore $\overline{y} = \frac{3(32)}{10(12)} = \frac{4}{5}$

(iii) When B is rotated about the y-axis the resulting volume is given by

$$\int_0^2 \pi x^2\,dy = \int_0^2 \pi y^6\,dy = \pi\left[\frac{y^7}{7}\right]_0^2 = \frac{128\pi}{7}$$

Hence the volume when A rotated is that of a cylinder $-$ the above volume, i.e.

$$(\pi 8^2)2 - \frac{128\pi}{7} = 128\pi - \frac{128\pi}{7} = 128\pi \times \frac{6}{7} = \frac{768\pi}{7}(= 344.68)$$