

**Vector Calculus**  
*Grad, Div and Curl Identities*

**Question**

It is given that  $\phi$  and  $\psi$  are scalar fields and  $\underline{F}$  and  $\underline{G}$  are vector fields. They are all assumed to be smooth functions. Prove the following identity

$$\nabla \bullet (\underline{F} \times \underline{G}) = (\nabla \times \underline{F}) \bullet \underline{G} - \underline{F} \bullet (\nabla \times \underline{G})$$

**Answer**

$$\begin{aligned} \nabla \bullet (\underline{F} \times \underline{G}) &= \frac{\partial}{\partial x} (F_2 G_3 - F_3 G_2 + \dots) \\ &= \frac{\partial F_2}{\partial x} G_3 + F_2 \frac{\partial G_3}{\partial x} - \frac{\partial F_3}{\partial x} G_2 - F_3 \frac{\partial G_2}{\partial x} + \dots \\ &= (\nabla \times \underline{F}) \bullet \underline{G} - \underline{F} \bullet (\nabla \times \underline{G}) \end{aligned}$$