

Vector Calculus
Grad, Div and Curl Identities

Question

It is given that ϕ and ψ are scalar fields and \underline{F} and \underline{G} are vector fields. They are all assumed to be smooth functions. Prove the following identity

$$\nabla \bullet (\phi \underline{F}) = (\nabla \phi) \bullet \underline{F} + \phi (\nabla \bullet \underline{F})$$

Answer

$$\begin{aligned} \nabla \bullet (\phi \underline{F}) &= \frac{\partial}{\partial x}(\phi F_1) + \frac{\partial}{\partial y}(\phi F_2) + \frac{\partial}{\partial z}(\phi F_3) \\ &= \frac{\partial \phi}{\partial x} F_1 + \phi \frac{\partial F_1}{\partial x} + \dots + \frac{\partial \phi}{\partial z} F_3 + \phi \frac{\partial F_3}{\partial z} + \dots \\ &= (\nabla \phi) \bullet \underline{F} + \phi (\nabla \bullet \underline{F}) \end{aligned}$$