

Vector Calculus
Grad, Div and Curl Identities

Question

Given that ϕ and ψ are smooth scalar fields, show that

$$\nabla \times (\phi \nabla \psi) = -\nabla \times (\psi \nabla \phi) = \nabla \phi \times \nabla \psi.$$

Answer

$$\begin{aligned}\nabla \times (\phi \nabla \psi) &= \nabla \phi \times \nabla \psi + \phi \nabla \times \nabla \psi \\ &= \nabla \phi \times \nabla \psi \\ -\nabla \times (\psi \nabla \phi) &= -\nabla \psi \times \nabla \phi - \psi \nabla \times \nabla \phi \\ &= \nabla \phi \times \nabla \psi.\end{aligned}$$