Vector Calculus
Grad, Div and Curl Identities

Question
If $\underline{F}$ is given by
$$\underline{F} = xe^{2z}\underline{i} + ye^{2z}\underline{j} - e^{2z}\underline{k},$$
show that $\underline{F}$ is a solenoidal vector field. Find a vecotr field for $\underline{F}$.
Answer
Given $\underline{F}$
$$\Rightarrow \text{div}\underline{F} = e^{2z} + e2z - 2e^{2z} = 0$$
so $\underline{F}$ is solenoidal.
If $\underline{F} = \nabla \times \underline{G}$
$$\begin{aligned} \Rightarrow \frac{\partial G_3}{\partial y} - \frac{\partial G_2}{\partial z} &= xe^{2z} \\ \frac{\partial G_1}{\partial z} - \frac{\partial G_3}{\partial x} &= ye^{2z} \\ \frac{\partial G_2}{\partial x} - \frac{\partial G_1}{\partial y} &= -e^{2z} \end{aligned}$$
Find a solution with $G_2 = 0$.
$$\Rightarrow G_3 = \int xe^{2z}\,dy = xye^{2z} + M(x,z).$$
Try setting $M(x,z) = 0$, $\Rightarrow G_3 = xye^{2z}$. So now
$$\begin{aligned} \frac{\partial G_1}{\partial z} &= ye^{2z} + \frac{\partial G_3}{\partial x} = 2ye^{2z} \\ \Rightarrow G_1 &= \in 2ye^{2z}\,dz = ye^{2z} + N(x,y) \\ \text{As } -e^{2z} &= = \frac{\partial G_1}{\partial y} = -e^{2z} - \frac{\partial N}{\partial y}, \\ \text{take } N(x,y) &= 0 \end{aligned}$$
So a vector potetnial for $\underline{F}$ is given by
$$\underline{G} = ye^{2z}\underline{i} + xye^{2z}\underline{k}.$$