Vector Calculus
Grad, Div and Curl Identities

Question

Find a vector potential for $\underline{F} = -y\underline{i} + x\underline{j}$.

Answer

$$\Rightarrow \frac{\partial G_3}{\partial y} - \frac{\partial G_2}{\partial z} = -y$$

$$\frac{\partial G_1}{\partial z} - \frac{\partial G_3}{\partial x} = x$$

$$\frac{\partial G_2}{\partial x} - \frac{\partial G_1}{\partial y} = 0$$

Find a solution with $G_2 = 0$

$$\Rightarrow G_3 = -\int y \, dy = -\frac{y^2}{2} + M(x, z).$$

Try setting $M(x, z) = 0$, $\Rightarrow G_3 = -\frac{y^2}{2}$. So now

$$\frac{\partial G_3}{\partial x} = 0$$

$$\text{and } G_1 = \int x \, dx = xz + N(x, y)$$

As $\frac{\partial G_1}{\partial y} = 0$, use $N(x, y) = 0$.

So a (non-unique) vector potential for $\underline{F}$ is given by

$$\underline{G} = xz\underline{i} - \frac{1}{2}y^2\underline{k}.$$