

**Vector Calculus**  
*Grad, Div and Curl Identities*

**Question**

It is given that  $\phi$  and  $\psi$  are scalar fields and  $\underline{F}$  and  $\underline{G}$  are vector fields. They are all assumed to be smooth functions. Prove the following identity

$$\nabla \times (\nabla\phi) = \underline{0}$$

**Answer**

By equality of mixed partials

$$\begin{aligned} \nabla \times (\nabla\phi) &= \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial\phi}{\partial x} & \frac{\partial\phi}{\partial y} & \frac{\partial\phi}{\partial z} \end{vmatrix} \\ &= \left( \frac{\partial}{\partial y} \frac{\partial\phi}{\partial z} - \frac{\partial}{\partial z} \frac{\partial\phi}{\partial y} \right) \underline{i} + \dots = \underline{0} \end{aligned}$$