Question

Show that $f(x) = |x-2|$ on the interval $[1, 4]$ satisfies neither the hypotheses nor the conclusion of the Mean Value Theorem.

Answer

First, note that f is continuous on $[1, 4]$, as it is the composition of two continuous functions, namely absolute value and a linear polynomial. However, f is not differentiable at $x = 2$ (since absolute value is not differentiable at 0), and so the hypotheses of the mean value theorem are not satisfied.

To see that f does not satisfy the conclusion of the mean value theorem, we calculate: $f(4)-f(1) = |4-2|-|1-2| = 2-1 = 1$ and $4-1 = 3$. However, for $x > 2$, we have that $f'(x) = 1$ and for $x < 2$ we have that $f'(x) = -1$, and so there cannot be a point c in $(1, 4)$ at which $f'(c) = (f(4)-f(1))/(4-1) = 1/3$.