Question
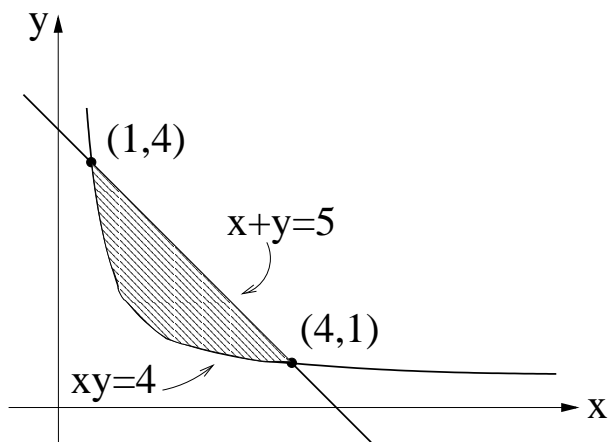
Sketch the region enclosed by the given curves and find the volume of the solid generated when it is revolved about the x-axis:

$xy = 4$, $x + y = 5$.

Answer

At which points does the line $x + y = 5$ intersect the curve $xy = 4$? Writing $y = 5 - x$ we have $xy = x(5 - x) = 4$ and so $5x - x^2 = 4$ or $x^2 - 5x + 4 = 0$. This factorises to $(x - 1)(x - 4) = 0$ and hence $x = 1$ or $x = 4$. Using $y = 5 - x$ we have the points $(x, y) = (1, 4)$ and $(x, y) = (4, 1)$.

Use washer method:

$$\int_{x=1}^{x=4} \left\{ \pi(5 - x)^2 - \pi \left(\frac{4}{x} \right)^2 \right\} dx$$

$$= \pi \int_{x=1}^{x=4} \left\{ (5 - x)^2 - \frac{16}{x^2} \right\} dx$$

$$= \pi \left[-\frac{(5 - x)^3}{3} + \frac{16}{x} \right]_{x=1}^{x=4}$$

$$= \pi \left\{ \left(-\frac{1}{3} + 4 \right) - \left(-\frac{4^3}{3} + 16 \right) \right\}$$

$$= 9\pi.$$