

Question

The lifetime X of an electrical component has an exponential distribution such that $P(X \leq 1000) = 0.75$. What is the expected lifetime of the component?

Answer

Given that $X \sim \text{exponential}(\beta)$ and $P\{X \leq 1000\} = 0.75$ then the problem is to find β since $E(X) = \beta$.

$$f(x|\beta) = \frac{1}{\beta} e^{-\frac{x}{\beta}}, \quad 0 < x < \infty$$

$$\text{For any } a > 0, P\{x \leq a\} = F(a) = \int_0^a \frac{1}{\beta} e^{-\frac{y}{\beta}} dy = 1 - e^{-\frac{a}{\beta}}$$

$$P\{X \leq 1000\} = 1 - e^{-\frac{1000}{\beta}} = 0.75 \Rightarrow \beta = 721.35$$