

Exam Question

Topic: Fourier Series

Find the Fourier Series for the function

$$f(x) = \pi - |x| \quad (-\pi \leq x \leq \pi).$$

Solution

f is an even function so $b_n = 0$ for all n .

$a_0 =$ area of triangle under the graph of $f = \pi$

$$\begin{aligned} a_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx = \frac{2}{\pi} \int_0^{\pi} (\pi - x) \cos nx \, dx \\ &= \frac{2}{\pi} \int_0^{\pi} (\pi \cos nx - x \cos nx) \, dx \\ &= \frac{2}{\pi} \left[\pi \frac{\sin nx}{n} - x \frac{\sin nx}{n} - \frac{\cos nx}{n^2} \right]_0^{\pi} \\ &= \frac{2}{\pi} \left[-\frac{\cos n\pi}{n^2} + \frac{1}{n^2} \right] = \frac{2}{n^2\pi} (1 - (-1)^n) \end{aligned}$$

So the Fourier Series is

$$\frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{2}{n^2\pi} (1 - (-1)^n) \cos nx$$