QUESTION A random sample of size n is taken from a population of size N. Write down the number of distinct samples when sampling is

(i) ordered, with replacement

(ii) ordered, without replacement

(iii) unordered, without replacement

(iv) unordered, with replacement

(n.b. (**iv**) is harder, it may help to write down a few special cases first).

ANSWER

(i) There is a choice of N for each therefore N^n

(ii) $N(N-1)(N-2)\ldots(N-n+1) = NP_n = \frac{N!}{(N_n)!}$

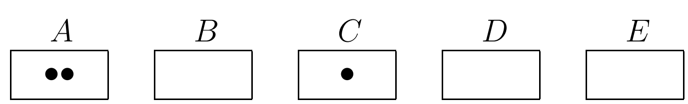
(iii) Each choice in this section goes to $n!$ samples in (ii), since each unordered sample can be ordered in $n!$ ways. Number of samples $= \frac{N!}{(N-n)!n!} = NC_n$

(iv) Example, $N = 5$ and $n = 3$.

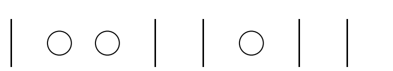
	AAA	AAB	ABC
	BBB	AAC	ABD
	CCC	AAD	ABE etc.
total number	5	20	$\binom{5}{3}$

Total $= 35 = \binom{7}{3} = \binom{N+n-1}{n}$ Problem corresponds first to the classical one of having N boxes and requiring the placing of n balls with any number in each box.

e.g.



This in turn corresponds to the problem of arranging $N+1$ lines and n circles in a row give that first and last must be a line.



corresponds to AAC. This can be done in $\binom{N+1-2+n}{n}$ ways (since the first and last are fixed.)