

QUESTION

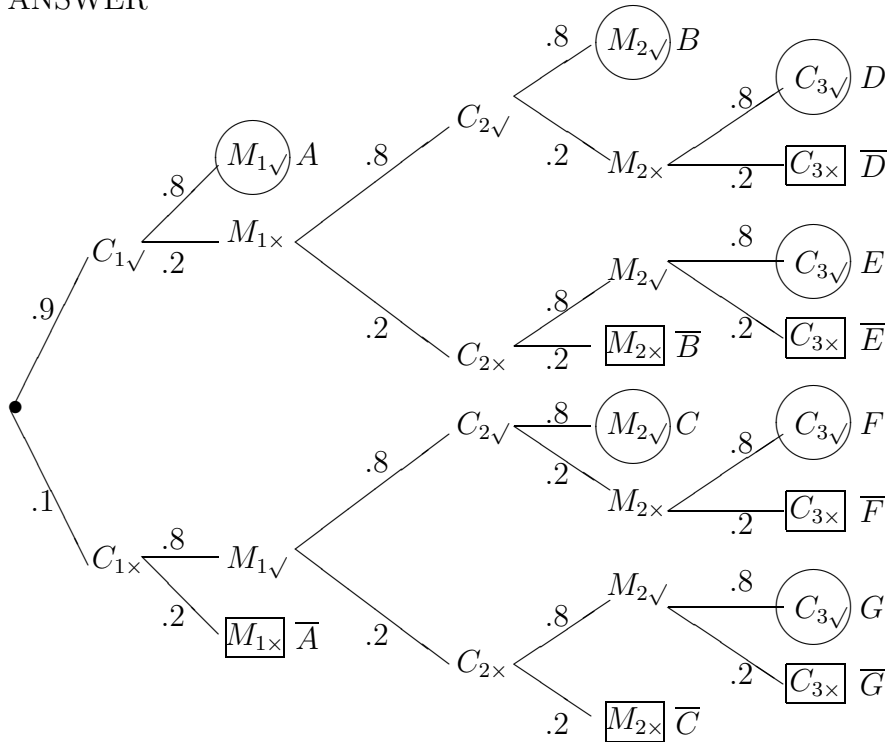
An electronic system receives signals as input and sends out appropriate coded messages as output.

The system consists of 3 converters C_1, C_2 and C_3 , 2 monitors M_1 and M_2 and a perfectly reliable three way switch for connecting the input to the converters. The incoming signal is changed into a code by one of the converters and the monitors check whether the conversion is correct



Initially the signal is fed into C_1 . If M_1 passes the conversion the coded message is sent out. If M_1 rejects the conversion the input is switched to C_2 and the conversion is checked by M_2 . If M_2 passes the conversion, the coded message is sent out. If M_2 rejects the conversion, the input is switched to C_3 and the coded message is sent out without any further checks.

Each of the converters has probability 0.9 of correctly coding the incoming message. Each of the monitors has probability 0.8 of rejecting a wrongly coded message and also probability 0.8 of passing a correctly coded message. Draw a probability tree and hence show that the probability of a correct output from the system is about 0.968.

ANSWER



$M_{1\times}$ M_1 monitor incorrect
 $M_{1\checkmark}$ M_1 monitor correct
 $C_{1\times}$ C_1 converter incorrect
 $C_{1\checkmark}$ C_1 converter correct

 correct outcome
 incorrect outcome

P correct
 A $0.9 \times 0.8 = 0.72$
 B $0.9^2 \times 0.2 \times 0.8 = 0.1296$
 C $0.9 \times 0.1 \times 0.8^2 = 0.0576$
 D $0.93 \times 0.2^2 = 0.02916$
 E $0.9^2 \times 0.2 \times 0.1 \times 0.8 = 0.01296$
 F $0.1 \times 0.8 \times 0.9^2 \times 0.2 = 0.1296$
 G $0.1^2 \times 0.8^2 \times 0.9 = 0.00576$
 Total = $0.96804 \approx 0.968$

P incorrect
 \bar{A} $0.1 \times 0.2 = 0.02$
 \bar{B} $0.9 \times 0.2^2 \times 0.1 = 0.0036$
 \bar{C} $0.1^2 \times 0.8 \times 0.2 = 0.0016$
 \bar{D} $0.92 \times 0.1 \times 0.2^2 = 0.00324$
 \bar{E} $0.9 \times 0.2 \times 0.12 \times 0.8 = 0.00144$
 \bar{F} $0.12 \times 0.8 \times 0.9 \times 0.2 = 0.00144$
 \bar{G} $0.1^3 \times 0.8^2 = 0.00064$
 Total as check = 0.03196