

**Question**

If  $f : R \rightarrow R$ ,  $g : R \rightarrow R$  etc. discuss the relationship between the two statements

- i)  $f$  is continuous a.e.
- ii) there is a continuous  $g$  such that  $f = g$  a.e.

**Answer**

- a) Let  $f = X_Q$  then if  $g = 0$ ,  $g = f$  a.e. So  $g$  is continuous, but  $f$  is continuous nowhere.
- b) Let  $f(x) = 1$ ,  $x \geq 0$ ,  $f$  is continuous a.e.  
If  $g = f$  a.e. Then there is a sequence  $x_n \rightarrow 0+$  such that  $f(x_n) = 1$ , and also a sequence  $y_n \rightarrow 0-$  such that  $g(y_n) = 0$ . Therefore  $g$  is not continuous at 0.