Question

Show that a function $f : \mathbf{R^n} \rightarrow R$ which is unbounded above and below cannot be represented as a monotonic limit of simple functions.

Answer

Let $\{f_n\}$ be an increasing sequence of simple monotonic functions.

Let $f_1 = \sum_{i=1}^{n} c_i X_{Ei}$ and let $c = \min_{i=1\cdots n} c_i$

Then for all $x \epsilon \mathbf{R^n}$ and for all $n \epsilon \mathbf{N}, \quad F_n(x) \geq c$

Hence if f is unbounded below we cannot have $f_n \rightarrow f$ everywhere. Similarly for decreasing sequences.