Question

In this question we consider the following matrices

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} \quad D = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

Consider the following products. In each case:

(i) establish whether the quantity exists and is well defined

(ii) if it does exist, evaluate it.

(a) BA (b) AC (c) BC (d) BC^T
(e) CA (f) $(CB)A$ (g) BD (h) CD

Answer

(a) BA : number of columns of $B = 3 \neq$ number of rows of $A = 2$

Therefore the product does not exist

(b) AC : number of columns of $A = 3 =$ number of rows of C

Therefore the product does exist and gives a two by two matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} = \begin{pmatrix} 1+6+15 & 2+8+18 \\ 4+15+30 & 8+20+36 \end{pmatrix} = \begin{pmatrix} 22 & 28 \\ 49 & 64 \end{pmatrix}$$

(c) BC : number of columns of $B = 3 =$ number of rows of C

Therefore the product does exist and gives a three by two matrix

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} = \begin{pmatrix} 1+3 & 2+4 \\ 5 & 6 \\ 3+5 & 4+6 \end{pmatrix} = \begin{pmatrix} 4 & 6 \\ 5 & 6 \\ 8 & 10 \end{pmatrix}$$

(d) B is 3×3; C^T is 2×3; BC^T does not exist

(e) CA : C is 3×2 and A is 2×3

Therefore the product does exist and gives a three by three matrix

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} = \begin{pmatrix} 1+9 & 2+10 & 3+12 \\ 3+16 & 6+20 & 9+24 \\ 5+24 & 10+20 & 15+36 \end{pmatrix} = \begin{pmatrix} 9 & 12 & 15 \\ 19 & 26 & 33 \\ 29 & 40 & 51 \end{pmatrix}$$

(f) C is 3×2; B is 3×3; therefore CB does not exist

(g) B is 3×3; D is 2×2; therefore BD does not exist

(h) C is 3×2; D is 2×2; therefore CD is a 3×2 matrix

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1+6 & 2+8 \\ 3+12 & 6+16 \\ 5+18 & 10+24 \end{pmatrix} = \begin{pmatrix} 7 & 10 \\ 15 & 22 \\ 23 & 34 \end{pmatrix}$$