Question

Write down the matrix of the minors M_{ij} and the matrix of cofactors A_{ij}

associated with the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 1 & 1 \end{pmatrix}$ with elements a_{ij}

Answer

$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 1 & 1 \end{pmatrix}$

Minors

$$[M_{ij}] = \begin{bmatrix} \begin{vmatrix} 5 & 6 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 4 & 6 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 4 & 5 \\ 1 & 1 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 1 & 1 \end{vmatrix} \\ \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} & \begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} & \begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix} \end{bmatrix}$$

$$= \begin{pmatrix} -1 & -2 & -1 \\ -1 & -2 & -1 \\ -3 & -6 & -3 \end{pmatrix}$$

Cofactors

$[A_{ij}] = \begin{pmatrix} -1 & 2 & -1 \\ 1 & -2 & 1 \\ -3 & 6 & -3 \end{pmatrix}$