Question

Find the scalar product $\mathbf{a} \cdot \mathbf{b}$ and hence find the angle between the vectors $\mathbf{a}$ and $\mathbf{b}$ given that:

(i) $\mathbf{a} = 7\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ $\mathbf{b} = -\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$

(ii) $\mathbf{a} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ $\mathbf{b} = -3\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$

(iii) $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ $\mathbf{b} = -2\mathbf{i} - 4\mathbf{j} - 6\mathbf{k}$

Answer

(i)

$$\begin{aligned} \mathbf{a} \cdot \mathbf{b} &= (7, -3, 1) \cdot (-1, 2, 2) \\ &= (7 \times -1) + (-3 \times 2) + (1 \times 2) \\ &= -7 - 6 + 2 = \underline{-11} \end{aligned}$$

Now $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$

$\Rightarrow \cos\theta = \dfrac{(\mathbf{a} \cdot \mathbf{b})}{|\mathbf{a}||\mathbf{b}|}$

So need

$|\mathbf{a}| = \sqrt{7^2 + (-3)^2 + 1^2} = \sqrt{59}$

$|\mathbf{b}| = \sqrt{(-1)^2 + 2^2 + 2^2} = \sqrt{9} = 3$

Therefore $\cos\theta = -\dfrac{11}{3\sqrt{59}} \Rightarrow \theta =$

(ii)

$$\begin{aligned} \mathbf{a} \cdot \mathbf{b} &= (-2\mathbf{i} - 2\mathbf{j} + \mathbf{k}) \cdot (-3\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}) \\ &= (2, -2, 1) \cdot (-3, -3, 4) \\ &= (2 \times -3) + (-2 \times -3) + (1 \times 4) \\ &= -6 + 6 + 4 = \underline{4} \end{aligned}$$

$|\mathbf{a}| = \sqrt{2^2 + (-2)^2 + 1^2} = \sqrt{9} = 3$

$|\mathbf{b}| = \sqrt{(-3)^2 + (-3)^2 + 4^2} = \sqrt{34}$

$\Rightarrow \cos\theta = \dfrac{(\mathbf{a} \cdot \mathbf{b})}{|\mathbf{a}||\mathbf{b}|} = \dfrac{4}{3\sqrt{34}} \Rightarrow \theta =$

(iii)

$$\begin{aligned} \mathbf{a}\cdot\mathbf{b} &= (\mathbf{i}+2\mathbf{j}+3\mathbf{k})\cdot(-2\mathbf{i}-4\mathbf{j}-6\mathbf{k}) \\ &= (1\times-2)+(2\times-4)+(3\times-6) \\ &= -2-8-18=-28 \end{aligned}$$

$|\mathbf{a}| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$

$|\mathbf{b}| = \sqrt{(2^2+4^2+6^2} = \sqrt{56}$

$\Rightarrow \cos\theta = \dfrac{(\mathbf{a}\cdot\mathbf{b})}{|\mathbf{a}||\mathbf{b}|} = \dfrac{-28}{\sqrt{14}}\sqrt{56} = -1 \Rightarrow \theta = \arccos(-1) = \underline{\pi}$

Could have spotted this from anti-parallel vectors.