## Question

Find the angle between the planes $2 x+6 y+z=4$ and $x+3 y+2 z=3$.
Answer
$2 x+6 y+z=4$ can be written:
$(x \mathbf{i}+y \mathbf{j}+z \mathbf{k}) \cdot(2 \mathbf{i}+6 \mathbf{j}+\mathbf{k})=4$
or $\mathbf{r} \cdot \mathbf{n}_{1}=D$ (1)
$\mathbf{r}=(x \mathbf{i}+y \mathbf{j}+z \mathbf{k}), \mathbf{n}_{1}=(2 \mathbf{i}+6 \mathbf{j}+\mathbf{k}), D=4$
Now we know $\mathbf{r} \cdot \hat{\mathbf{n}}_{1}=d(2)$ is the vector equation of a plane with unit normal $\hat{\mathbf{n}}_{1}$. Thus comparing (1) and (2) we have with $\hat{\mathbf{n}}_{1}\left|\mathbf{n}_{1}\right|=\mathbf{n}_{1}, \frac{d}{|\mathbf{n}|}=d$. In any case $\mathbf{n}_{1}$ is normal to the plane $2 x+6 y+z=4$. Similarly $\mathbf{n}_{2}=\mathbf{i}+3 \mathbf{j}+2 \mathbf{k}$ is normal to the plane $x+3 y+2 z=3$.
The angle between two planes is given by $\theta$, PICTURE
where $\mathbf{n}_{1} \cdot \mathbf{n}_{2}=\left|\mathbf{n}_{1}\right|\left|\mathbf{n}_{2}\right| \cos \theta$
Thus

$$
\begin{aligned}
\cos \theta & =\frac{(2,6,1) \cdot(1,3,2)}{\sqrt{2^{2}+6^{2}+1^{2}} \sqrt{1^{2}+3^{2}+2^{2}}} \\
& =\frac{2+18+2}{\sqrt{41} \sqrt{14}} \\
& =\frac{22}{\sqrt{41} \sqrt{14}} \\
\Rightarrow \theta & =\arccos \left(\frac{22}{\sqrt{41} \sqrt{14}}\right)
\end{aligned}
$$

