## Question

Find the angle between the planes 2x + 6y + z = 4 and x + 3y + 2z = 3.

## Answer

 $\begin{aligned} &2x + 6y + z = 4 \text{ can be written:} \\ &(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}) \cdot (2\mathbf{i} + 6\mathbf{j} + \mathbf{k}) = 4 \\ &\text{or } \mathbf{r} \cdot \mathbf{n}_1 = D \quad (1) \\ &\mathbf{r} = (x\mathbf{i} + y\mathbf{j} + z\mathbf{k}), \ \mathbf{n}_1 = (2\mathbf{i} + 6\mathbf{j} + \mathbf{k}), \ D = 4 \\ &\text{Now we know } \mathbf{r} \cdot \hat{\mathbf{n}}_1 = d \quad (2) \text{ is the vector equation of a plane with unit} \\ &\text{normal } \hat{\mathbf{n}}_1. \text{ Thus comparing (1) and (2) we have with } \hat{\mathbf{n}}_1 |\mathbf{n}_1| = \mathbf{n}_1, \ \frac{d}{|\mathbf{n}|} = d. \\ &\text{In any case } \mathbf{n}_1 \text{ is normal to the plane } 2x + 6y + z = 4. \text{ Similarly } \mathbf{n}_2 = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k} \\ &\text{ is normal to the plane } x + 3y + 2z = 3. \end{aligned}$ 

where  $\mathbf{n}_1 \cdot \mathbf{n}_2 = |\mathbf{n}_1| |\mathbf{n}_2| \cos \theta$ Thus

$$\cos \theta = \frac{(2,6,1) \cdot (1,3,2)}{\sqrt{2^2 + 6^2 + 1^2}\sqrt{1^2 + 3^2 + 2^2}} \\ = \frac{2 + 18 + 2}{\sqrt{41}\sqrt{14}} \\ = \frac{22}{\sqrt{41}\sqrt{14}} \\ \Rightarrow \theta = \arccos\left(\frac{22}{\sqrt{41}\sqrt{14}}\right)$$