QUESTION A man travels to work by train and bus. His train is due to arrive at 08.45 and the bus he hopes to catch is due at 08.48. The time of arrival of the train has a normal distribution with mean 08.44 and standard deviation 2 mins; the departure time of the bus is independently normally distributed with standard deviation 1 minute. Calculate the probabilities that:

(i) the train is late,

(ii) the bus departs before the train arrives,

(iii) in a period of 5 days there are at least 3 days on which the bus departs before the train arrives.

ANSWER

(i) The train is late if it arrives after 08.45, i.e. more than one minute after the mean. P(Late) $= 1 - \Phi(\frac{1}{2}) = 0.3085$

(ii) $T \sim N(08.44, 2^2), \quad B \sim N(08.48, 1^2)$
$P(B < T) = P(B - T < 0)$
$B - T \sim N(4, 5)$ therefore $P(B < T) = \Phi(\frac{0-4}{\sqrt{5}}) = 1 - \Phi(1.79) = .0365$

(iii) Number of days on which the bus departs before the train arrives B(5,0.0365)
P(Number of days ≥ 3) $= \begin{pmatrix} 5 \\ 3 \end{pmatrix} (0.0365)^3(0.9635)^2 + \begin{pmatrix} 5 \\ 4 \end{pmatrix} (0.0365)^4(0.9635) +$
$(0.0365)^5 = 0.0005$