QUESTION Given the continuous pdf $f(x) = \frac{2}{x^2},\ 1 \le x \le 2$, determine the mean and variance of x and find the probability that x exceeds 1.5. Calculate also the median and the quartile range for x and state the interquartile range $(Q_3 - Q_1)$.

ANSWER $f(x) = \frac{2}{x^2}\ 1 \le x \le 2$

$$\begin{aligned} \mu &= \int_1^2 x\frac{2}{x^2}\,dx = \int_1^2 \frac{2}{x}\,dx \\ &= [2\ln x]_1^2 = 2\ln 2 \end{aligned}$$

$$\begin{aligned} E(X^2) &= \int_1^2 x^2\frac{2}{x^2}\,dx = \int_1^2 2\,dx \\ &= [2x]_1^2 = 2 \\ \sigma^2 &= 2 - (2\ln 2)^2 \end{aligned}$$

$F(x) = \int_1^x \frac{2}{u^2}\,du = [-\frac{2}{u}]_1^x = 2 - \frac{2}{x}$

$P(X \le 1.5) = 1 - F(1,5) = 1 - (2 - \frac{2}{1.5}) = 1 - \frac{2}{3} = \frac{1}{3}$

Median M: $F(M) = 2 - \frac{2}{m} = \frac{1}{2}$ therefore $\frac{2}{m} = \frac{3}{2},\ M = \frac{4}{3}$

Quartile $Q_1:\ F(Q_!) = 2 - \frac{2}{Q_1} = \frac{1}{4}$ therefore $\frac{2}{Q_1} = \frac{7}{2},\ Q_1 = \frac{8}{7}$

Quartile $Q_3:\ F(Q_3) = 2 - \frac{2}{Q_3} = \frac{3}{4}$ therefore$\frac{2}{Q_3} = \frac{5}{4},\ Q_3 = \frac{8}{5}$

Interquartile range $= Q_3 - Q_1 = \frac{8}{5} - \frac{8}{7} = \frac{16}{35}$