Question

(i) $\int_0^{\frac{\pi}{2}} \sin x \, dx$

(ii) $\int_0^{\pi} \sin x \, dx$

(iii) $\int_0^{2\pi} \sin x \, dx$

(iv) $\int \sin^2 x \, dx$

(v) $\int \cos^2 x \, dx$

(vi) $\int_0^{2\pi} \sin^2 x \, dx$

Answer

(i)

$\int_0^{\frac{\pi}{2}} \sin x \, dx = [-\cos x]_0^{\frac{\pi}{2}} = -\cos\frac{\pi}{2} + \cos 0 = -0 + 1 = \underline{1}$

(ii)

$\int_0^{\pi} \sin x \, dx = [-\cos x]_0^{\pi} = -\cos\pi + \cos 0 = -(-1) + 1 = \underline{2}$

(iii)

$\int_0^{2\pi} \sin x \, dx = [-\cos x]_0^{2\pi} = -\cos 2\pi + \cos 0 = -1 + 1 = \underline{0}$

Pictures of (i), (ii) and (iii): PICTURE

(iv)

$\int \sin^2 x \, dx$: Now, $1 - 2\sin^2 x = \cos 2x \Rightarrow \sin^2 x = \frac{1}{2}(1 - \cos 2x)$

Therefore

$$\begin{aligned}\int \sin^2 x\,dx &= \frac{1}{2}(1+\cos 2x)\,dx\\ &= \frac{1}{2}\int dx + \frac{1}{2}\int \cos 2x\,dx\\ &= \underline{\frac{x}{2} - \frac{1}{4}\sin 2x + c}\end{aligned}$$

(standard power and standard multiple sine/cosine)
(v)
$\int \cos^2 x\,dx$:
Now, $2\cos^2 x - 1 = \cos 2x \Rightarrow \cos^2 x = \frac{1}{2}(1+\cos 2x)$
Therefore

$$\begin{aligned}\int \cos^2 x\,dx &= \frac{1}{2}\int dx + \frac{1}{2}\int \cos 2x\,dx\\ &= \underline{\frac{x}{2} + \frac{1}{4}\sin 2x + c}\end{aligned}$$

(compare with (iv) above)
(vi)

$$\begin{aligned}\int_0^{2\pi} \sin^2 x\,dx &= \left[\frac{x}{2} - \frac{1}{4}\sin 2x\right]_0^{2\pi} \quad \text{[from } x\text{]}\\ &= \left(\frac{2\pi}{2} - \frac{1}{4}\sin 4\pi\right) - (0-0) = \pi\end{aligned}$$

NB $\int_0^{\pi} \sin^2 x\,dx = 0$, but $\int_0^{2\pi} \sin^2 x\,dx = \pi \neq 0$
Why?
Plot graph of $\sin^2 x$: PICTURE