

Question

Determine whether the following matrices are singular or non-singular

$$A = \begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \quad D = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 1 & 2 \\ 2 & 1 & 6 \end{pmatrix}$$

Answer

Matrix A

$$\begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 2 \times 6 - 3 \times 4 = 12 - 12 = 0$$

Hence $\begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix}$ is singular.

Matrix B

$$\begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} = (-1)^{1+2} \times 1 \times \begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} = 0$$

Hence $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ is singular.

Matrix C

$$\begin{vmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{vmatrix} = (-1)^{1+1} \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} + (-1)^{1+4} \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix}$$

First determinant is zero since one row is completely zero.

Hence

$$\begin{aligned} \begin{vmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{vmatrix} &= (-1)^{1+4} \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{vmatrix} \\ &= -1 \times \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} \\ &= -1 \times -1 \\ &= 1 \end{aligned}$$

Hence $\begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$ is not singular.

Matrix D

$$\begin{aligned} \begin{vmatrix} 2 & 1 & 3 \\ 1 & 1 & 2 \\ 2 & 1 & 6 \end{vmatrix} &= \begin{vmatrix} 0 & 0 & -3 \\ 1 & 1 & 2 \\ 2 & 1 & 6 \end{vmatrix} && \text{row 1} \rightarrow \text{row 1} - \text{row 3} \\ &= \begin{vmatrix} 0 & 0 & -3 \\ 0 & 1 & 2 \\ 1 & 1 & 6 \end{vmatrix} && \text{column 1} \rightarrow \text{column 1} - \text{column 2} \\ &= 3 \end{aligned}$$

Hence $\begin{pmatrix} 2 & 1 & 3 \\ 1 & 1 & 2 \\ 2 & 1 & 6 \end{pmatrix}$ is not singular.