**Question**

Consider the Henon map with $b = 0.4$. Verify the following facts:

**(i)** For $-0.09 < a < 0.27$ there is one sink fixed point and one saddle fixed point.

**(ii)** As $a$ increases through 0.27, the largest magnitude eigenvalue of the first fixed point passes through -1, and an attracting 2-cycle is created.

**(iii)** This 2-cycle ceases to be attracting as $a$ increases through 0.85.

**Answer**

**(i)** Fixed points $(x, y$ given by $x^2 + (1 - b)x - a = 0,\ y = x$. Eigenvalues of $Df(y)$ given by $\lambda^2 + 2x\lambda - 6 = 0,\ \lambda = -x \pm \sqrt{x^2 + b}$: real as $b >$

As $a$ increases from -0.09 to 0.27 the x-coordinate of the fixed points spread out from -0.3 (repeated) to $x = -0.9,\ +0.3$.

For $x < -0.3$ we have $x^2 + b > 0.49$ so $+\sqrt{x^2 + b} > 0.7$ and therefore on eigenvalue $\lambda > 1$: saddle (the other eigenvalue stays inside the unit circle).

For $-0.3 < x < 0.3$ we have $x^2 + b < 0.49$ so both $|\lambda| < 1$: sink.

**(ii)** As $x$ increases through 0.3 (that is $a$ increases through 0.27) we see $\lambda = -x - \sqrt{x^2 + b}$ decreases through- We expect this to lead to creation of a 2-cycle, but check:

2-cycle is created as a increases through

$\frac{3}{4}(1 - b)^2 = \frac{3}{4}(0.6)^2 = 0.27.\checkmark$

Moreover this occurs at point (x,x) where $x = \frac{1}{2}(1 - b) = 0.3.\checkmark$

The eigenvalues of $Df^2$ at a per-2 point are $\lambda = t \pm \sqrt{t^2 - b^2}$ where (with $b = 0.4$) we have $t = 1.12 - 2a$. For a just $> 0.27$ we have $t < 0.58$ and $t^2 - b^2 < 0.1764$ so $+\sqrt{t^2 - b^2} < 0.42$: thus $\lambda = t + \sqrt{t^2 - b^2}$ just¡1, so the 2-cycle is attracting.

**(iii)** As $a$ increases thourgh 0.85 we see that:

$t$ decreases through -0.58

$\sqrt{t^2 - b^2}$ increases through 0.42.

So the eigenvalue $\lambda = t - \sqrt{t^2 - b^2}$ decreases through -1; 2-cycle no longer attracting.