Question

For each of the following matrices A decide whether the origin is a sink, source or saddle for the linear map $v \mapsto Av$. If a saddle, then draw the stable and unstable manifolds(lines):

(a) $\begin{pmatrix} 4 & 30 \\ 1 & 3 \end{pmatrix}$, (b) $\begin{pmatrix} 1 & \frac{1}{2} \\ \frac{1}{4} & \frac{3}{4} \end{pmatrix}$, (c) $\begin{pmatrix} -0.4 & 2.4 \\ -0.4 & 1.6 \end{pmatrix}$

Answer

(a) Eigenvalues: $(\lambda-4)(\lambda-3)-30 = 0$ i.e. $\lambda^2-7\lambda-18 = 0$: $(\lambda-9)(\lambda+2) = 0$. Thus $\lambda = 9, -2$: both outside unit circle. Therefore <u>(flip) source</u>.

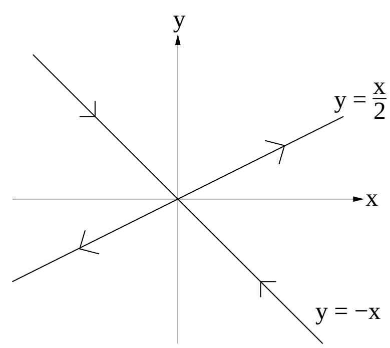
(b) Eigenvalues: $(\lambda-1)(\lambda-\frac{3}{4}) - \frac{1}{8} = 0$ i.e. $\lambda^2 - \frac{7}{4}\lambda + \frac{5}{8} = 0$:

$(\lambda-\frac{5}{4})(\lambda-\frac{1}{2}) = 0$. Thus $\lambda = \dfrac{5}{4}, \dfrac{1}{2}$: <u>saddle</u> (no flip).

Eigenvectors:

$\lambda = \dfrac{5}{4}$ $\begin{pmatrix} -\frac{1}{4} & \frac{1}{2} \\ \frac{1}{4} & -\frac{1}{2} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ say.

$\lambda = \dfrac{1}{2}$ $\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$: $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ say.



(c) Eigenvalues: $(\lambda+0.4)(\lambda-1.6) + 0.96 = 0$ i.e. $\lambda^2 - 1.2\lambda + 0.32 = 0$: $(\lambda-0.4)(\lambda-0.8) = 0$.

Thus $\lambda = 0.4, 0.8$: both inside the unit circle. Therefore <u>sink</u> (no flip).