

QUESTION

Evaluate $\int_C z^m \bar{z}^n dz$ where C is the unit circle $|z| = 1$, and m and n are integers.

ANSWER

If we parametrize the unit circle C as $z = e^{it}$, ($0 \leq t \leq 2\pi$), then we get $\int_C z^m \bar{z}^n dz = i \int_0^{2\pi} e^{(m+1-n)it} dt = \frac{i}{(m+1-n)i} [e^{(m+1-n)it}]_0^{2\pi} = 0$ if $m+1 \neq n$, and if $m+1 = n$ we get $2\pi i$.