Question

Show that

i) $T - (S_1 \cap S_2) = (T - S_1) \cup [(T \cap S_1) - S_2]$

ii) $S_1 \cup S_2 = [(S_1)^C \cap (S_2)^C]^C$

Answer

i) $(T - S_1) \cup [(T \cap S_1) - S_2]$

$= (T \cap (S_1)^C) \cup (T \cap S_1 \cap (S_2)^C)$

$= [(T \cap (S_1)^C) \cup T] \cap [(T \cap (S_1)^C) \cup S_1] \cap [T \cap (S_1)^C \cup (S_2)^C]$

(Distributive law)

$= T \cap [T \cup S_1] \cap [T \cup (S_2)^C] \cap [(S_1)^C \cup (S_2)^C]$

(using $A \cup (A \cap B) = A$ and $(A - B) \cup B = A \cup B$)

$= T \cap (S_1 \cap S_2)^C$

(using $A \cap (A \cup B) = A$ twice and De-Morgan's Law)

$= T - (S_1 \cap S_2)$

ii) $(S_1 \cup S_2)^C = (S_1)^C \cap (S_2)^C$ by De-Morgan

Therefore $S_1 \cup S_2 = [(S_1)^C \cap (S_2)^C]^C$ (note $((S)^C)^C = S$)