Question

Find the mean and the rms values of $f(t) = \cos\omega t$ over the half period $0 \leq y \leq \frac{\pi}{\omega}$ and over the full period $0 \leq t \leq \frac{2\pi}{\omega}$.

Answer

Mean over $\frac{1}{2}$ period:

$$\begin{aligned}\overline{f_{\frac{1}{2}}} &= \frac{1}{(\frac{\pi}{\omega}-0)}\int_0^{\frac{\pi}{\omega}} dt\cos\omega t\\ &= \frac{\omega}{\pi}\left(\frac{\sin\omega 2}{\omega}\right]_0^{\frac{\pi}{\omega}} = \underline{0}\end{aligned}$$

Since we have $\cos\underline{\omega t}$ a full period is $\dfrac{2\pi}{\omega}$

Mean over full period:

$$\begin{aligned}\overline{f_{full}} &= \frac{1}{(\frac{2\pi}{\omega}-0)}\int_0^{\frac{2\pi}{\omega}} dt\cos\omega t\\ &= \frac{\omega}{2\pi}\left(\frac{\sin\omega t}{\omega}\right]_0^{\frac{2\pi}{\omega}} = \underline{0}\end{aligned}$$

RMS value over $\frac{1}{2}$ period:

$$\begin{aligned}f_{rms,\frac{1}{2}} &= \sqrt{\frac{1}{\frac{\pi}{\omega}-0}\int_0^{\frac{\pi}{\omega}} dt\cos^2\omega t}\\ &= \sqrt{\frac{\omega}{\pi}\frac{1}{2}\int_0^{\frac{\pi}{\omega}} dt(1+\cos 2\omega t)}\\ &= \left[\frac{\omega}{2\pi}\left[t+\frac{\sin 2\omega t}{2\omega}\right]_0^{\frac{\pi}{\omega}}\right]^{\frac{1}{2}}\\ &= \left(\frac{\omega}{2\pi}\cdot\frac{\pi}{\omega}\right)^{\frac{1}{2}}\\ &= \underline{\frac{1}{\sqrt{2}}}\end{aligned}$$

RMS value over full period:

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\begin{aligned}
f_{rms,full} &= \sqrt{\frac{1}{\frac{2\pi}{\omega} - 0} \int_0^{\frac{\pi}{\omega}} dt \cos^2 \omega t} \\
&= \sqrt{\frac{\omega}{2\pi} \times \frac{1}{2} \left[t + \frac{\sin 2\omega t}{2\omega} \right]_0^{\frac{2\pi}{\omega}}} \\
&= \underline{\frac{1}{\sqrt{2}}}
\end{aligned}
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