Question

In another circuit, the applied voltage is $V = 100\sin\omega t + 16\sin 3\omega t$ volts. The corresponding current is $I = 5.8\sin\left(\omega t - \frac{\pi}{3}\right) + 1.6\sin\left(3\omega t - \frac{\pi}{6}\right)$ amps. Find the mean and rms values of the power P dissipated over a period.

Answer

$I = 5\cdot 8\sin\left(\omega t - \frac{\pi}{3}\right) + 1\cdot 6\sin\left(3\omega t - \frac{\pi}{6}\right)$

$V = 100\sin\omega t + 16\sin 3\omega t$

$$P = IV = \begin{array}{rl} & 580\sin\omega t\sin\left(\omega t - \frac{\pi}{3}\right) \\ + & 160\sin\omega t\sin\left(3\omega t - \frac{\pi}{6}\right) \\ + & 92\cdot 8\sin 3\omega t\sin\left(\omega t - \frac{\pi}{3}\right) \\ + & 25\cdot 6\sin 3\omega t\sin\left(3\omega t - \frac{\pi}{6}\right) \end{array}$$

What is the period? It's the longest times for any of the series to repeat. The most slowly varying sine is $\sin\omega t \Rightarrow$ period is $t = \frac{2\pi}{\omega}$.

$$\bar{P} = \frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}} P\,dt$$

This is a complicated integral. Expand the sines using the addition formula

$$\sin(A - B) = \sin A\cos B - \cos A\sin B$$