

Question

Explain **exactly** what is meant by the following statements:

1. $\lim_{x \rightarrow 1} (2x)^4 = 16$;
2. $\lim_{x \rightarrow -3} (3x^2 + e^x) = 81 + e^{-3}$;

Answer

(Note that we are not asked to determine whether the statement is correct or not, and if it is correct we are not asked to prove it. This is an exercise in writing down the definition of $\lim_{x \rightarrow a} f(x) = L$ for specific values of a and L and a specific function $f(x)$.)

1. For every $\varepsilon > 0$, there exists $\delta > 0$ so that if $0 < |x - 1| < \delta$, then $|(2x)^4 - 16| < \varepsilon$.
2. For every $\varepsilon > 0$, there exists $\delta > 0$ so that if $0 < |x - (-3)| = |x + 3| < \delta$, then $|(3x^2 + e^x) - (81 + e^{-3})| < \varepsilon$.