

Partial Differentiation
Functions of more than one variable

Question

For the given functions $f(x, y, z)$, describe the level surfaces.

(a) $f(x, y, z) = x^2 + y^2 + z^2$

(b) $f(x, y, z) = x + 2y + 3z$

(c) $f(x, y, z) = x^2 + y^2$

(d) $f(x, y, z) = \frac{x^2 + y^2}{z^2}$

(e) $f(x, y, z) = |x| + |y| + |z|$

Answer

(a) $f(x, y, z) = x^2 + y^2 + z^2$

The level surface $f(x, y, z) = c > 0$ is a sphere of radius \sqrt{c} centred at the origin.

(b) $f(x, y, z) = x + 2y + 3z$

The level surfaces are parallel planes with common normal vector $\underline{i} + 2\underline{j} + 3\underline{k}$.

(c) $f(x, y, z) = x^2 + y^2$

The level surface $f(x, y, z) = c > 0$ is a circular cylinder of radius \sqrt{c} with axis along the z -axis.

(d) $f(x, y, z) = \frac{x^2 + y^2}{z^2}$

The equation $f(x, y, z) = c$ can be rewritten $x^2 + y^2 = C^2 z^2$. The level surfaces are circular cones with vertices at the origin and axes along the z -axis.

(e) $f(x, y, z) = |x| + |y| + |z|$

The level surface $f(x, y, z) = c > 0$ is the surface of the octohedron with vertices $(\pm c, 0, 0)$, $(0, \pm c, 0)$ and $(0, 0, \pm c)$. (An octohedron is a solid with eight planar faces.)