QUESTION

A company manufactures batches of electrical components, with each batch containing 10 components. Past experience shows that 80% of batches contain one defective component and 20% of batches contain five defective components. The following options are available for each batch.

(i) Send the batch to the next stage of production without inspection of components.

(ii) Reprocess the batch at a cost of £1000 and then send it to the next stage of production. A reprocessed batch contains one defective component.

(iii) Test two components from the batch at a cost of £150. Use the evidence of the test, either to send the batch immediately to the next stage of production, or to reprocess the batch first and then send it.

Each defective component which is sent to the next stage of production costs £500. Which option would you recommend?
Suggest how the cost might be reduced by following an alternative policy.

ANSWER

Let G denote a good batch (1 defective component) and B a bad batch (5 defective components. Then $P(G) = 0.8$ and $P(B) = 0.2$.
Let T_0, T_1 and T_2 be the outcomes of the test.

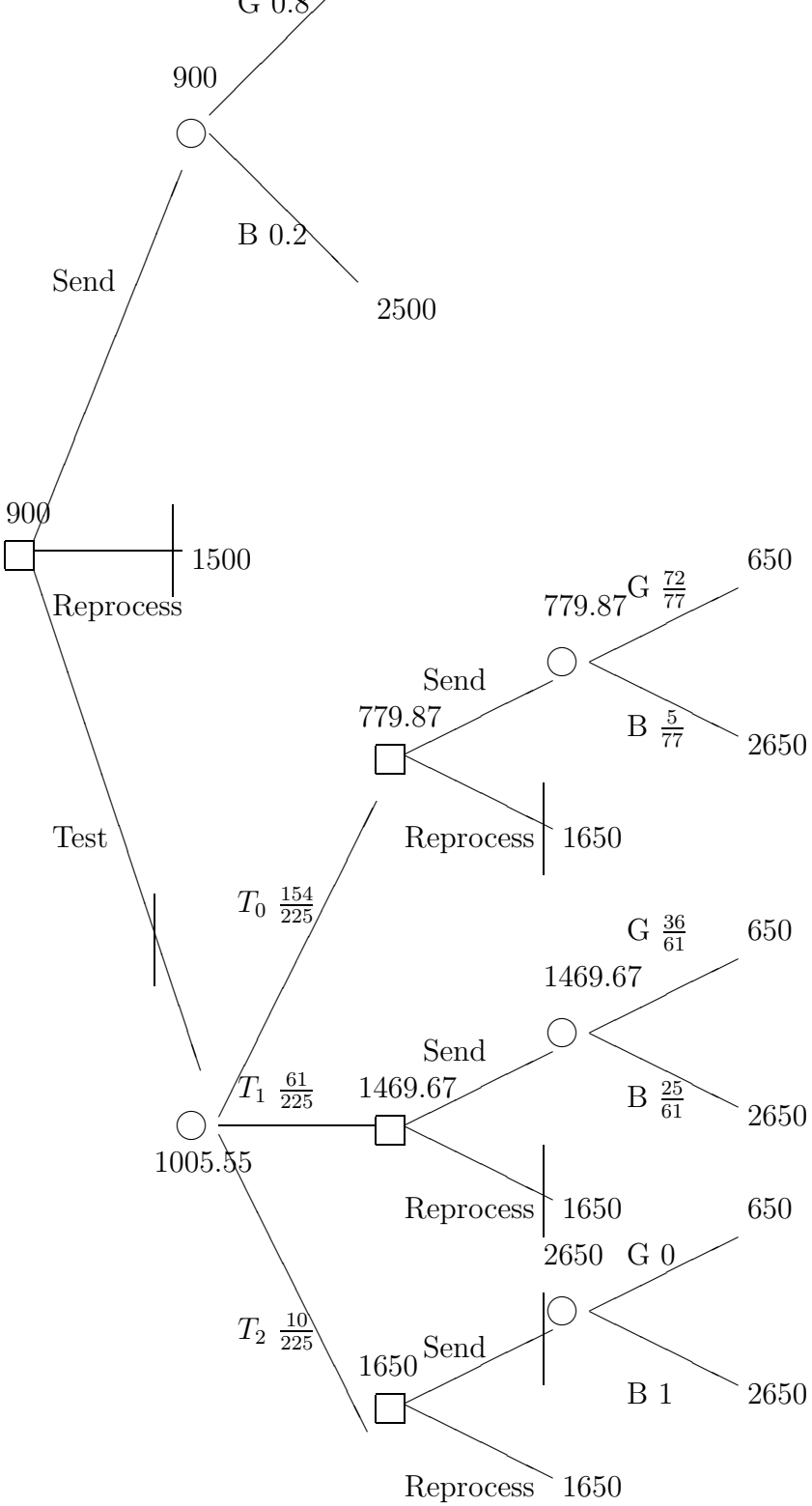
$$
\begin{aligned}
P(T_0|G) &= \frac{\binom{9}{2}}{\binom{10}{2}} = 0.8 \\
P(T_0|B) &= \frac{\binom{5}{2}}{\binom{10}{2}} = \frac{2}{9} \\
P(T_1|G) &= \frac{\binom{9}{1}\binom{1}{1}}{\binom{10}{2}} = 0.2 \\
P(T_1|B) &= \frac{\binom{5}{1}\binom{5}{1}}{\binom{10}{2}} = \frac{5}{9} \\
P(T_2|G) &= 0
\end{aligned}
$$

$$P(T_2|B) = \frac{\binom{5}{2}}{\binom{10}{2}} = \frac{2}{9}$$

$$\begin{aligned}
P(T_0) &= P(T_0|G)P(G) + P(T_0|B)P(B) = \frac{154}{225} \\
P(T_1) &= P(T_1|G)P(G) + P(T_1|B)P(B) = \frac{61}{225} \\
P(T_2) &= P(T_2|G)P(G) + P(T_2|B)P(B) = \frac{10}{225}
\end{aligned}$$

$$\begin{aligned}
P(G|T_0) &= \frac{P(G \cap T_0)}{P(T_0)} = \frac{P(T_0|G)P(G)}{P(T_0)} = \frac{72}{77} \\
P(B|T_0) &= \frac{P(B \cap T_0)}{P(T_0)} = \frac{P(T_0|B)P(B)}{P(T_0)} = \frac{5}{77} \\
P(G|T_1) &= \frac{P(G \cap T_1)}{P(T_1)} = \frac{P(T_1|G)P(G)}{P(T_1)} = \frac{36}{61} \\
P(B|T_1) &= \frac{P(B \cap T_1)}{P(T_1)} = \frac{P(T_1|B)P(B)}{P(T_1)} = \frac{25}{61} \\
P(G|T_2) &= \frac{P(G \cap T_2)}{P(T_2)} = \frac{P(T_2|G)P(G)}{P(T_2)} = 0 \\
P(B|T_2) &= \frac{P(B \cap T_2)}{P(T_2)} = \frac{P(T_2|B)P(B)}{P(T_2)} = 1
\end{aligned}$$

G is a good batch (1 defective component)
B is a bad batch (5 defective components)
T_0 the test yields 0 defective components
T_1 the test yields 1 defective component
t_2 the test yields 2 defective components



Prior probabilities are $P(G) = 0.8$ and $P(B) = 0.2$. The recommended option is to send to the next stage, without inspection. It might be cheaper to test only one component.