Question

Find the general solution of the differential equation

$$\frac{dx}{dt} + 3x = e^{-2t}$$

Answer

$$\frac{dx}{dt} + 3x = e^{-2t}$$

Compare with the model equation $\dfrac{dx}{dt} + p(t)x = q(t)$
Multiply by integrating factor $g(t) = e^{\int p(t)\,dt}$ so that $\frac{d}{dt}[xg(t)] = g(t)q(t)$
Here the integrating factor is $g(t) = e^{\int 3\,dt} = e^{3t}$

$$\begin{aligned}
\text{Thus} \qquad e^{3t}\frac{dx}{dt} + 3e^{3t}x &= e^{3t}e^{-2t} = e^{3t-2t} = e^{t} \\
\text{Hence} \qquad \frac{d}{dt}(xe^{3t}) &= e^{t} \\
\text{Finally} \qquad xe^{3t} &= \int e^{t}\,dt = e^{t} + C \\
\Rightarrow x &= e^{-2t} + Ce^{-3t}
\end{aligned}$$

Where C is a constant.