Question

Find the solution of the initial value problem

$$\frac{dx}{dt} + \frac{3}{t}x = t - \frac{2}{t^2}$$

given $x(1) = 1$

Answer

$$\frac{dx}{dt} + \frac{3}{t}x = t - \frac{2}{t^2}$$

with $x(1) = 1$ for $t > 1$

Here the integrating factor is $e^{\int \frac{3}{t}\,dt} = e^{3ln|t|} = |t|^3 = t^3$ for $t > 1$

Thus $t^3 \dfrac{dx}{dt} + 3t^2 x = t^4 - 2t$

Hence $\dfrac{d}{dt}(t^3 x) = t^4 - 2t \Rightarrow xt^3 = \dfrac{1}{5}t^2 - \dfrac{1}{t} + \dfrac{C}{t}$

Where C is a constant.

Find C using initial condition $1 = \dfrac{1}{5} - 1 + C \Rightarrow C = \dfrac{9}{5} \Rightarrow x = \dfrac{1}{5}t^2 - \dfrac{1}{t} + \dfrac{9}{5t^3}$