Exam Question
Topic: Double Integral
Let I denote the repeated integral

$$\int_0^1 dx \int_0^{2x-x^2} \exp\left[(1-y)^{3/2}\right]\, dy.$$

Reverse the order of integration and hence evaluate I. Give your answer in terms of e, and also as a decimal correct to 3 places.

Solution
Now $y = 2x - x^2 \Rightarrow x = 1 \pm \sqrt{1-y}$. But $0 \le x \le 1$ and so $x = 1 - \sqrt{1-y}$. So reversing the order of integration gives

$$\begin{aligned} I &= \int_0^1 dy \int_{1-\sqrt{1-y}}^1 \exp\left[(1-y)^{3/2}\right]\, dx = \int_0^1 \left[x \exp(1-y)^{3/2}\right]_{1-\sqrt{1-y}}^1 \\ &= \int_0^1 \sqrt{1-y}\, \exp\left[(1-y)^{3/2}\right]\, dy = \left[\tfrac{2}{3}(-1)\exp(1-y)^{3/2}\right]_0^1 \\ &= \tfrac{2}{3}(-1+\mathrm{e}) = 1.146 \quad 3 \text{ d.p.} \end{aligned}$$