

Question

Given that

$$\begin{vmatrix} 5 & 4 & 7 \\ 7 & 2 & 2 \\ 6 & 3 & 1 \end{vmatrix} = 63$$

Find without expansion

$$(i) \begin{vmatrix} 50 & 4 & 7 \\ 70 & 2 & 2 \\ 60 & 3 & 1 \end{vmatrix}$$

$$(ii) \begin{vmatrix} 15 & 20 & -56 \\ 21 & 10 & -16 \\ 18 & 15 & -8 \end{vmatrix}$$

$$(iii) \begin{vmatrix} 5x^3 & \frac{2}{x} & 21y^2 \\ 7x^3 & \frac{1}{x} & 6y^2 \\ 6x^3 & \frac{3}{2x} & 3y^2 \end{vmatrix}$$

Answer

$$\begin{vmatrix} 5 & 4 & 7 \\ 7 & 2 & 2 \\ 6 & 3 & 1 \end{vmatrix} = 63$$

We will use that multiplication of 1 row/column by k scales the determinant value by k .

(i)

$$\begin{aligned} \begin{vmatrix} 50 & 4 & 7 \\ 70 & 2 & 2 \\ 60 & 3 & 1 \end{vmatrix} &= \begin{vmatrix} 10 \times 5 & 4 & 7 \\ 10 \times 7 & 2 & 2 \\ 10 \times 6 & 3 & 1 \end{vmatrix} \\ &= 10 \begin{vmatrix} 5 & 4 & 7 \\ 7 & 2 & 2 \\ 6 & 3 & 1 \end{vmatrix} \\ &= 10 \times 63 \\ &= \underline{630} \end{aligned}$$

(ii)

$$\begin{aligned} \begin{vmatrix} 15 & 20 & -56 \\ 21 & 10 & -16 \\ 18 & 15 & -8 \end{vmatrix} &= \begin{vmatrix} 3 \times 5 & 5 \times 4 & -8 \times 2 \\ 3 \times 6 & 5 \times 3 & -8 \times 1 \\ 3 \times 6 & 5 \times 3 & -8 \times 1 \end{vmatrix} \\ &= 3 \times \begin{vmatrix} 5 & 5 \times 4 & -8 \times 7 \\ 7 & 5 \times 2 & -8 \times 2 \\ 6 & 5 \times 3 & -8 \times 1 \end{vmatrix} \\ &= 3 \times 5 \times \begin{vmatrix} 5 & 4 & -8 \times 7 \\ 7 & 2 & -8 \times 2 \\ 6 & 3 & -8 \times 1 \end{vmatrix} \\ &= 3 \times 5 \times -8 \times \begin{vmatrix} 5 & 4 & 7 \\ 7 & 2 & 2 \\ 6 & 3 & 1 \end{vmatrix} \\ &= 3 \times 5 \times -8 \times 63 \\ &= \underline{-7560} \end{aligned}$$

(iii)

$$\begin{aligned} \begin{vmatrix} 5x^3 & \frac{2}{x} & 21y^2 \\ 7x^3 & \frac{1}{x} & 6y^2 \\ 6x^3 & \frac{3}{2x} & 3y^2 \end{vmatrix} &= x^3 \begin{vmatrix} 5 & \frac{2}{x} & 21y^2 \\ 7 & \frac{1}{x} & 6y^2 \\ 6 & \frac{3}{2x} & 3y^2 \end{vmatrix} \\ &= x^3 \times \frac{1}{x} \times \begin{vmatrix} 5 & 2 & 21y^2 \\ 7 & 1 & 6y^2 \\ 6 & \frac{3}{2} & 3y^2 \end{vmatrix} \\ &= x^3 \times \frac{1}{x} \times y^2 \times \begin{vmatrix} 5 & 2 & 21 \\ 7 & 1 & 6 \\ 6 & \frac{3}{2} & 3 \end{vmatrix} \\ x^2 y^2 \begin{vmatrix} 5 & \frac{1}{2} \times 4 & 21 \\ 7 & \frac{1}{2} \times 2 & 6 \\ 6 & \frac{1}{2} \times 3 & 3 \end{vmatrix} &= x^2 y^2 \times \frac{1}{2} \begin{vmatrix} 5 & 4 & 3 \times 7 \\ 7 & 2 & 3 \times 2 \\ 6 & 3 & 3 \times 1 \end{vmatrix} \\ &= \frac{3}{2} x^2 y^2 \begin{vmatrix} 5 & 4 & 7 \\ 7 & 2 & 2 \\ 6 & 3 & 1 \end{vmatrix} \\ &= \frac{3}{2} x^2 y^2 \times 63 \\ &= \underline{\underline{\frac{189}{2} x^2 y^2}} \end{aligned}$$