

Question

$$(i) \begin{vmatrix} 4 & 6 & 7 \\ 2 & 3 & 8 \\ 1 & 9 & 0 \end{vmatrix}$$

$$(ii) \begin{vmatrix} 2 & -3 & 0 \\ -4 & 5 & 1 \\ 0 & 6 & 8 \end{vmatrix}$$

$$(iii) \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

Answer

(i)

$$\begin{vmatrix} 4 & 6 & 7 \\ 2 & 3 & 8 \\ 1 & 9 & 0 \end{vmatrix} \\ = 4 \begin{vmatrix} 3 & 8 \\ 9 & 0 \end{vmatrix} - 6 \begin{vmatrix} 2 & 8 \\ 1 & 0 \end{vmatrix} + 7 \begin{vmatrix} 2 & 3 \\ 1 & 9 \end{vmatrix}$$

$$NB: \begin{vmatrix} \alpha & \beta \\ \gamma & \delta \end{vmatrix} = \alpha\delta - \beta\gamma$$

Using minors, expanding by the 1st row

and remembering the $+ - +$ sign pattern.

$$\begin{aligned} &= 4(3 \times 0 - 9 \times 8) - 6(2 \times 0 - 8 \times 1) + 7(2 \times 9 - 1 \times 3) \\ &= 4(0 - 72) - 6(0 - 8) + 7(18 - 3) \\ &= -288 + 48 + 105 \\ &= \underline{\underline{-135}} \end{aligned}$$

(ii)

$$\begin{aligned} & \begin{vmatrix} 2 & -3 & 0 \\ -4 & 5 & 1 \\ 0 & 6 & 8 \end{vmatrix} \\ &= 2 \begin{vmatrix} 5 & 1 \\ 6 & 8 \end{vmatrix} - (-3) \begin{vmatrix} -4 & 1 \\ 0 & 8 \end{vmatrix} + 0 \begin{vmatrix} -4 & 5 \\ 0 & 6 \end{vmatrix} \\ & \text{Using minors, expanding by the 1st row} \\ & \text{and remembering the } + - + \text{ sign pattern.} \\ &= 2(40 - 6) + 3(-32 - 0) + 0(-24 - 0) \\ &= 78 - 96 + 0 \\ &= \underline{-18} \end{aligned}$$

(iii)

$$\begin{aligned} & \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} \\ &= 1 \begin{vmatrix} 5 & 6 \\ 8 & 9 \end{vmatrix} - 2 \begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} + 3 \begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix} \\ & \text{Using minors, expanding by the 1st row} \\ & \text{and remembering the } + - + \text{ sign pattern.} \\ &= 1(45 - 48) - 2(36 - 42) + 3(32 - 35) \\ &= -3 + 12 - 9 \\ &= \underline{0} \end{aligned}$$

Note that we could have evaluated (iii) in the following way:

$$\begin{aligned} \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} &= \begin{vmatrix} 1 & 2 & 3 \\ 3 & 3 & 3 \\ 7 & 8 & 9 \end{vmatrix} \text{ Rule 4 : with row 2 - row 1} \\ & \text{(i.e., multiple = } -1 \times \text{ row (1))} \\ &= 3 \begin{vmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 7 & 8 & 9 \end{vmatrix} \text{ Rule 4 : with } k = 3 \\ &= 3 \begin{vmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} \text{ Rule 4 : with row 3 - 7} \times \text{ row 2} \\ & \text{(i.e., multiple = } -7 \times \text{ row (2))} \\ &= 0 \text{ Rule 2 : row 1 \& row 3 are } \underline{\text{identical}} \end{aligned}$$