

QUESTION From the formulas for sin and cos in terms of the exponential function, prove that $\sin^2 z + \cos^2 z = 1$, for all complex numbers z .

ANSWER $\sin^2 z + \cos^2 z = \left(\frac{e^{iz} - e^{-iz}}{2i}\right)^2 + \left(\frac{e^{iz} + e^{-iz}}{2}\right)^2 = 1$ after expanding.