

Vector Fields
Conservative Fields

Question

For the following vector field, find whether it is conservative. If so, find a corresponding potential

$$\underline{F}(x, y) = \frac{x\underline{i} - y\underline{j}}{x^2 + y^2}$$

Answer

$$\begin{aligned} F_1 &= \frac{x}{x^2 + y^2} \\ F_2 &= -\frac{y}{x^2 + y^2} \\ \Rightarrow \frac{\partial F_1}{\partial y} &= -\frac{2xy}{(x^2 + y^2)^2} \\ \frac{\partial F_2}{\partial x} &= \frac{2xy}{(x^2 + y^2)^2} \end{aligned}$$

So \underline{F} cannot be conservative.